Gravitational Coleman-Weinberg Corrections to the Standard Model Higgs at Planck Scales

Shinshu U: Masaatsu Horikoshi

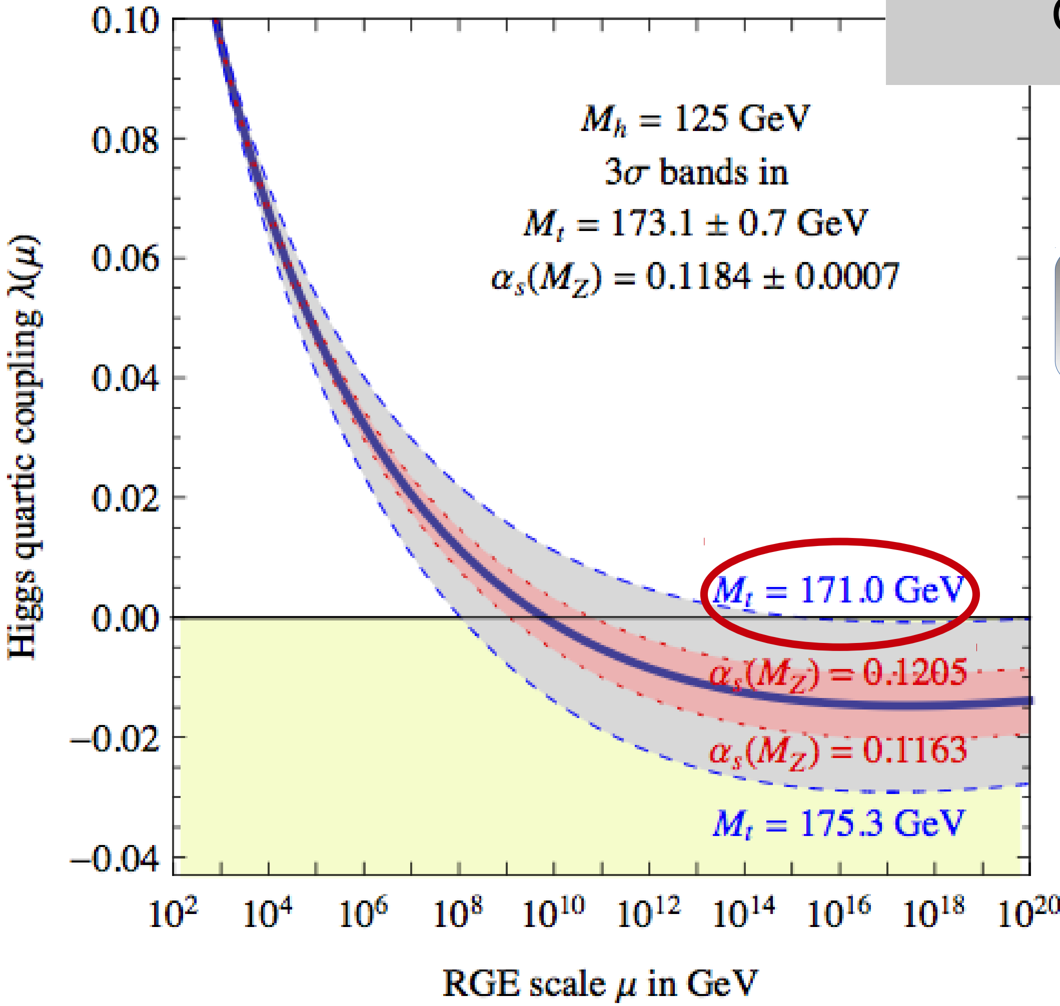
Collaborator
Taiwan U : Takeo Inami
Shinshu U : Yugo Abe

Introduction

Particles

Standared model	Beyond the SM
Complete	Undiscovered

Higgs vacuum stability



G. Degrassi , S. Di Vita, J.Elias-Miro , et. al. 2012

SM is valid up to Planck scales ?

Quantum gravitational effect should not be ignored .

Set up

Action

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R+\mathcal{L}_{gh}+g^{\mu\nu}(\partial_\mu H)^\dagger(\partial_\nu H)+m^2H^\dagger H-\lambda(H^\dagger H)^2+\mathcal{L}_{Gauge}+\mathcal{L}_{Fermi}\right]$$

Adding the gravitational term to the SM

Higgs potential (tree level)

$$V_{tree} = -\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4$$

metric

$$g_{\mu\nu}=\eta_{\mu\nu}+\frac{\sqrt{32\pi}}{M_{Pl}}h_{\mu\nu}$$

Gauge fixing(de・donder gauge)

$$\mathcal{L}_{gh} = -\eta_{\alpha\beta}\left(\eta^{\mu e}\eta^{\nu\alpha}-\frac{1}{2}\eta^{\mu\nu}\eta^{e\alpha}\right)\left(\eta^{\rho f}\eta^{\sigma\beta}-\frac{1}{2}\eta^{\rho\sigma}\eta^{f\beta}\right)h_{\mu\nu,e}h_{\rho\sigma,f}$$

Calculate the gravitational corrections

Gravitational 1-loop effect

$$
\begin{aligned}
V_{loop} \;=\; & \frac{3}{64\pi^2}\left(-m^2+\lambda\phi^2\right)^2\left(\ln\frac{-m^2+\lambda\phi^2}{\Lambda^2}-\frac{3}{2}\right) \\
& + \frac{36}{M_{Pl}^4}\left(-\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4\right)^2\left(\ln\left(\frac{16}{M_{Pl}^2}\frac{-\frac{1}{2}m^2\phi^2+\frac{1}{4}\lambda\phi^4}{\Lambda^2}\right)-\frac{3}{2}\right) \\
& + \sum_{i=\pm}\frac{C_i^2}{64\pi^2}\left(\ln\frac{C_i}{\Lambda^2}-\frac{3}{2}\right)
\end{aligned}
$$

$$
C_{\pm} \;=\; \frac{1}{2}\left\{V''_{tree}-\frac{16}{M_{Pl}^2}V_{tree}\pm\sqrt{\left(V''_{tree}+\frac{16}{M_{Pl}^2}V_{tree}\right)^2-\frac{128}{M_{Pl}^2}{V'_{tree}}^2}\right\}
$$

F. Loebbert and J. Plefka 2015

Condition of stability

~~λ>0~~

Existance of higher terms : Φ^6 , Φ^8 ➡

Evaluate the effective potential

Counter terms

Couplings of Φ^6 , Φ^8

These values are unkown

Set up

Φ^2 , Φ^4	Φ^6 , Φ^8
renormalizable	Non renormalizable

Φ^6 and Φ^8 terms depend on cut off

Counter terms

$$V_{counter} = A\, ln\frac{\Lambda^2}{\mu^2}\cdot\phi^2 + B\, ln\frac{\Lambda^2}{\mu^2}\cdot\phi^4$$

Effective potential

Annalyze the behavior

$$V_{eff} = V_{tree} + V_{loop} + V_{counter}$$

Analysis

$$V_{eff} = V_{tree} + V_{loop} + V_{counter}$$

Renormalization

Beta functions

Top yukawa,
gauge couplings,
Φ^4 coupling,
Φ^2 coupling

Corrections

- Gravitational : 1-loop
- SM's : 2-loop

Analysis

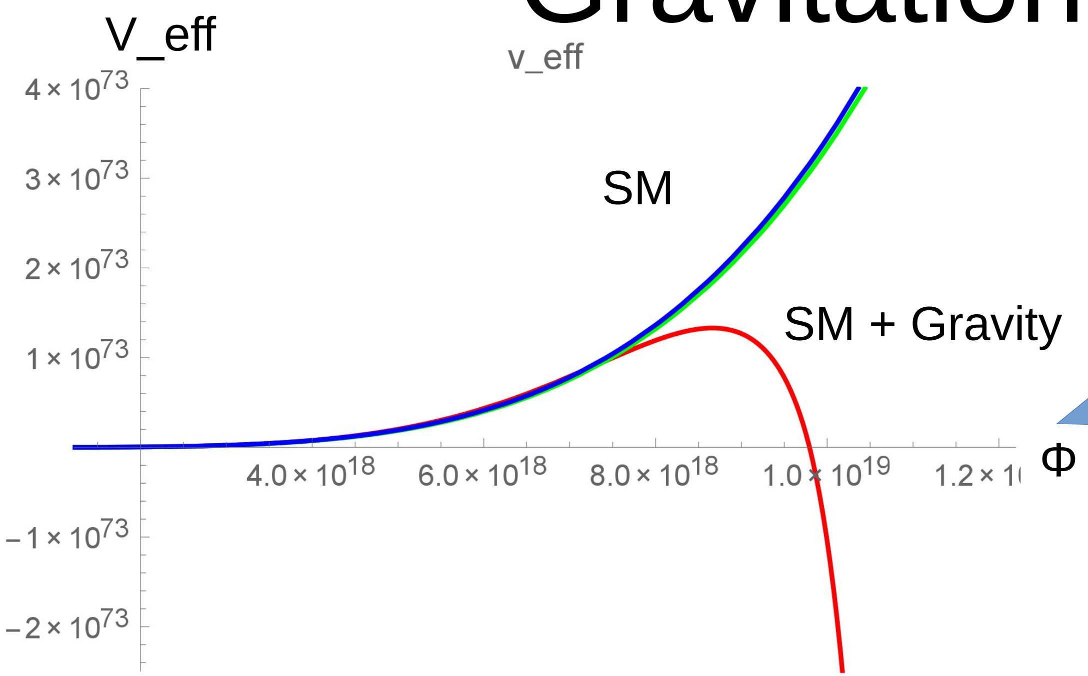
Difference

SM — SM+ Gravity

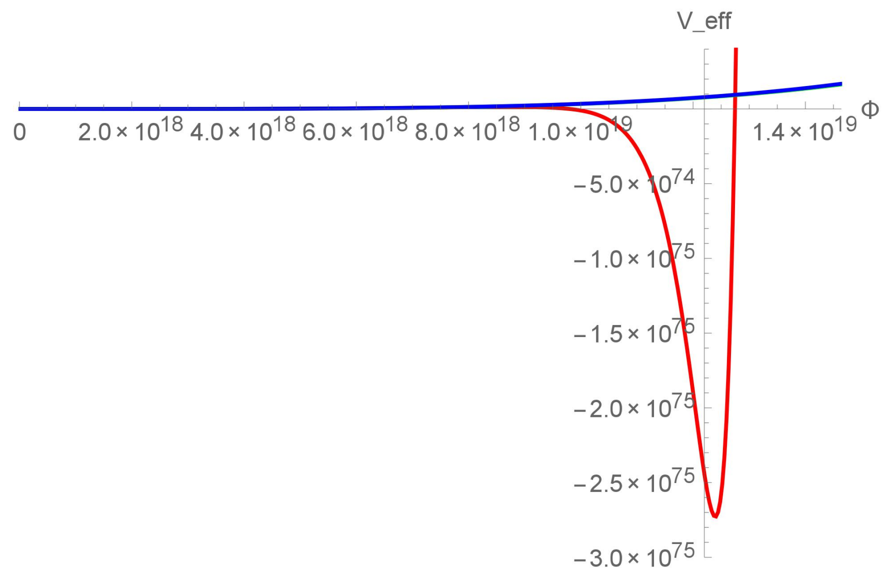
Dependance

- Cut off
- Top mass
- Higgs mass

Gravitational effect

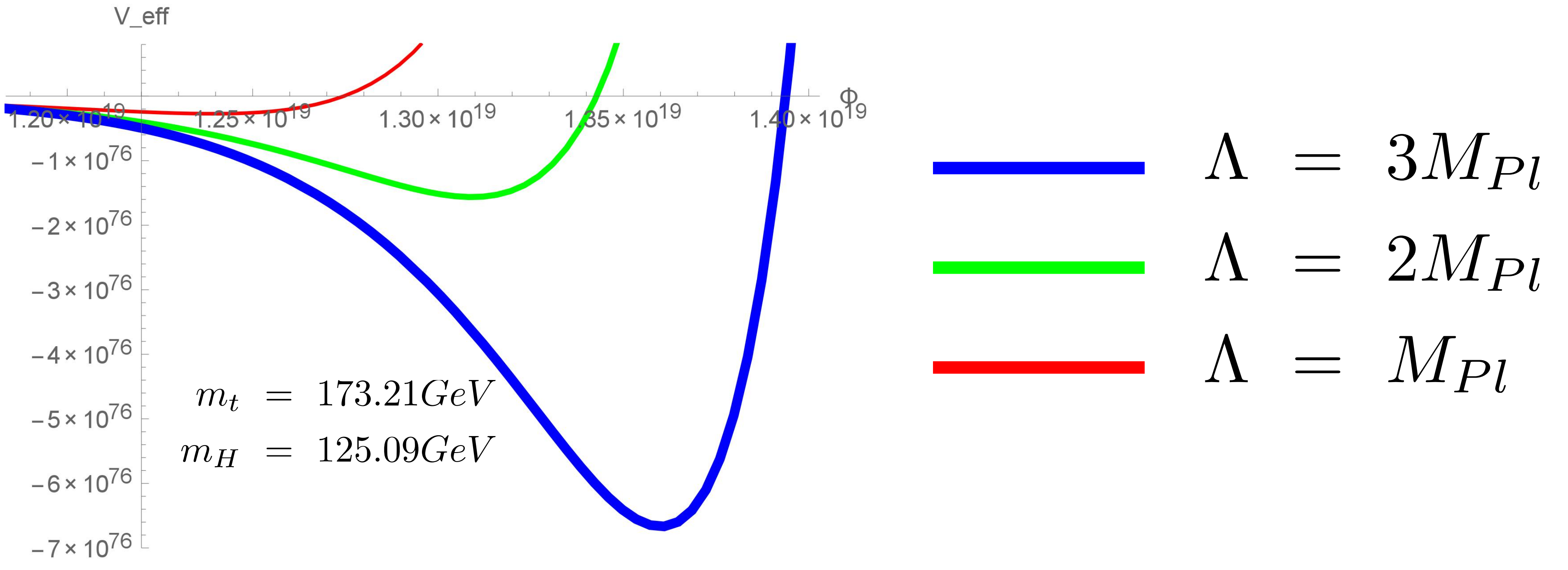


Gravitational correction is dominant from about 0.6 × Planck mass scales



Minimum at Planck scales

Cut off dependance

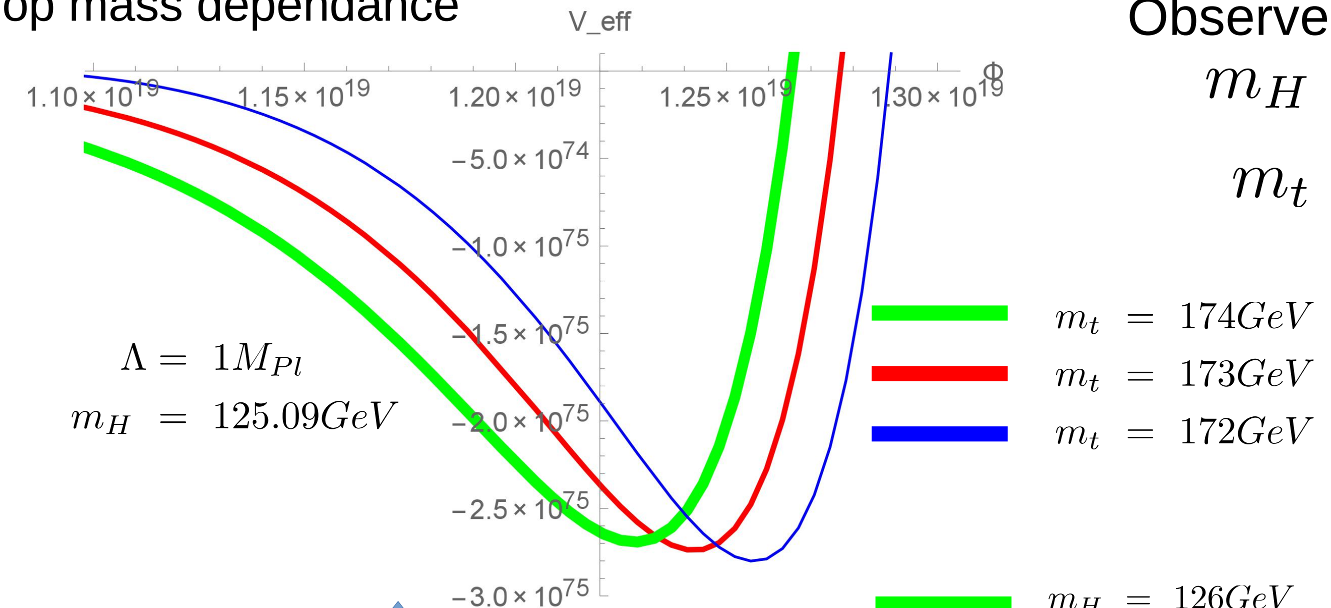


Value of Higgs field at the vacuum become large as we increase the cut off value.

The growth is not so big

Top& Higgs mass dependance

Top mass dependance

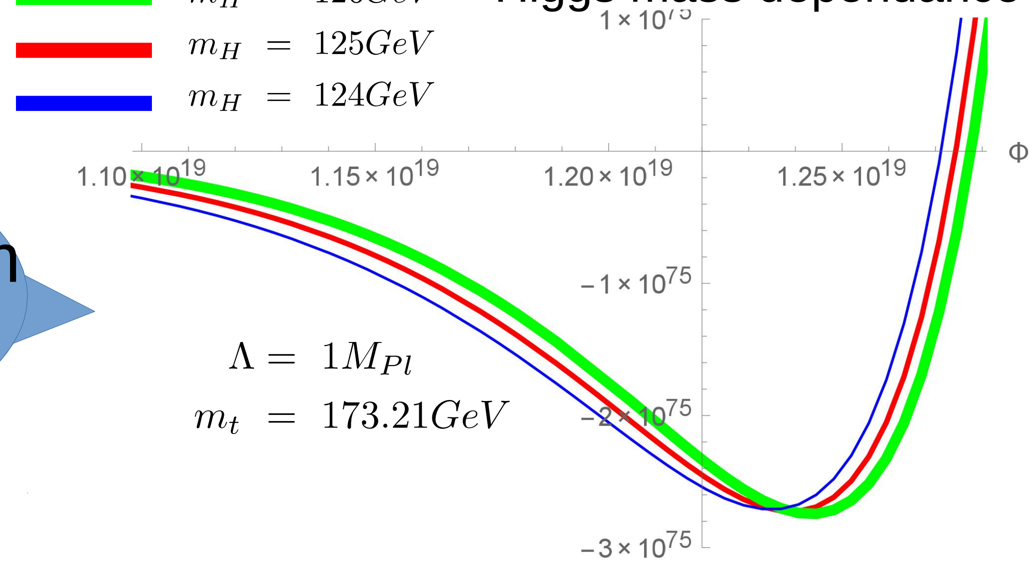


Observed value

$$m_H = 125.09 \pm 0.24$$

$$m_t = 173.21 \pm 0.51$$

Higgs mass dependance



Value of Higgs field at minimum stay arround Planck scales

Summary

- We evaluate the quantum gravitational corrections to Higgs potential.
- Quantum Gravitational corrections become effective from about $M_{Pl}/2$.
- Gravitational corrections to Higgs potential make minimum at Planck scales.

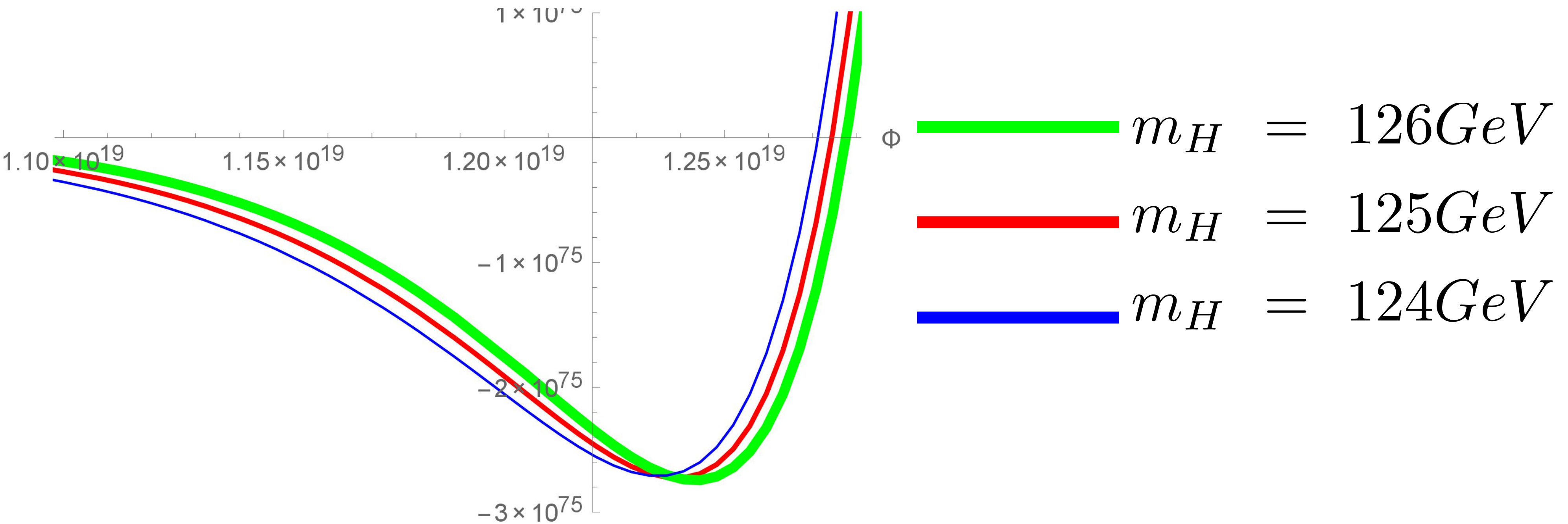
Higher terms

:we can ignore

More higher terms

We ignore to simplify

Higgs mass dependance



Analysis

$$V_{eff} = V_{tree} + V_{loop} + V_{counter}$$

Difference

SM — SM+ Gravity

Observed value

$$m_H = 125.09 \pm 0.24$$
$$m_t = 173.21 \pm 0.51$$

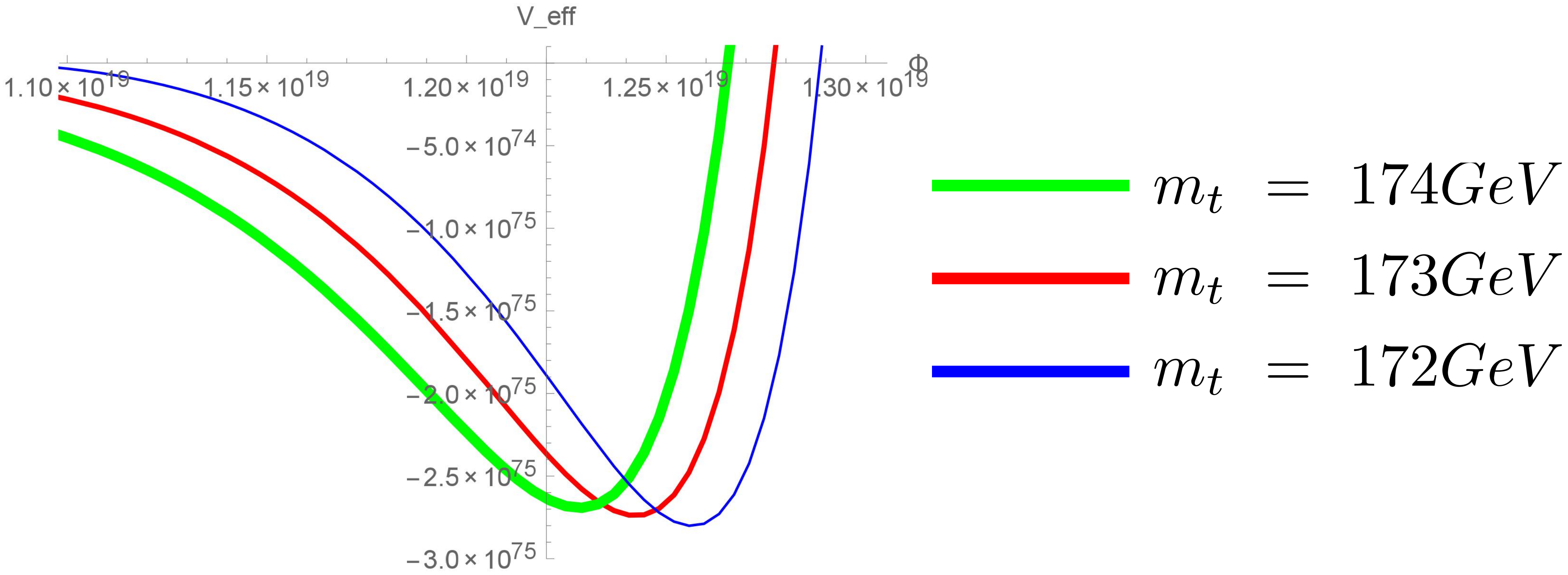
Dependance

- Cut off
- Top mass
- Higgs mass

Scope of analysis

$$\Lambda = 1 \sim 3M_{Pl}$$
$$m_t = 173 \pm 1GeV$$
$$m_H = 125 \pm 1GeV$$

Top mass dependance



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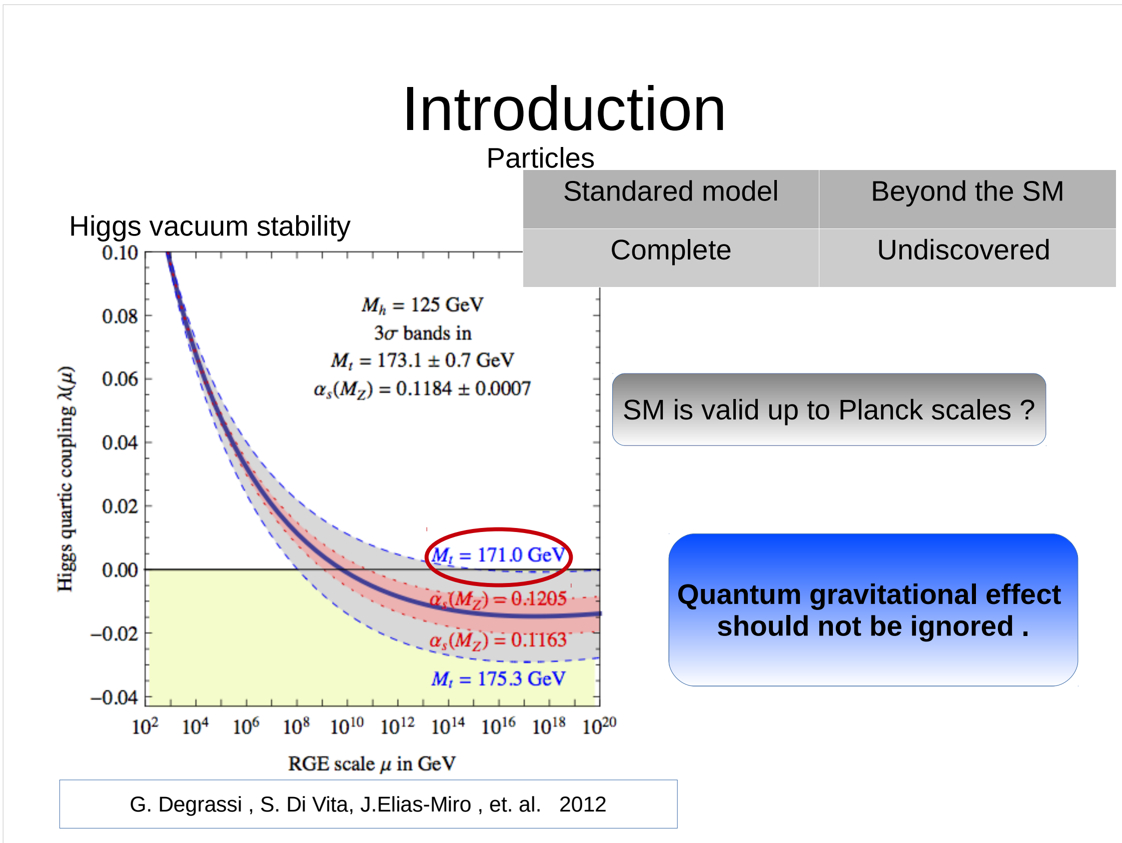
I would like to thank the organizers for giving me an opportunity to talk here.

Iam going to talk about the gravitational effect for the standared model Higgs's potential .

This study is based on the work with professer Inami, and Abe



2012, Higgs partcle was discovered at LHC .
And Standared model particles were completed.

On the other hand , any beyond SM particles are not discovered.

So there is a question , How far is the standard model valid?

Higgs vacuum stability give us an answer.

If we consider the stability with using corrections up to two loop's, there is a possibility that we can use SM all the way up to Planck scales.

This was suggested by Degrassi et. al.

On the other hand , We can not ignor gravitational corrections at planck scales

Purpose of this work is to analyze the gravitational effect.

Set up

Action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R + \mathcal{L}_{gh} + g^{\mu\nu}(\partial_\mu H)^\dagger(\partial_\nu H) + m^2 H^\dagger H - \lambda(H^\dagger H)^2 + \mathcal{L}_{Gauge} + \mathcal{L}_{Fermi}\right]$$

Adding the gravitational term to the SM

Higgs potential (tree level)

$$V_{tree} = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\sqrt{32\pi}}{M_{Pl}}h_{\mu\nu}$$

Gauge fixing(de･donder gauge)

$$\mathcal{L}_{gh} = -\eta_{\alpha\beta}\left(\eta^{\mu e}\eta^{\nu\alpha} - \frac{1}{2}\eta^{\mu\nu}\eta^{e\alpha}\right)\left(\eta^{\rho f}\eta^{\sigma\beta} - \frac{1}{2}\eta^{\rho\sigma}\eta^{f\beta}\right)h_{\mu\nu,e}h_{\rho\sigma,f}$$

Calculate the gravitational corrections

First I would like to talk about set up.

We consider the standard model ,copled Einstein's gravity.

Then the action is like this. We simply add the Einstein gravity to the SM.
And Now we ignore the cosmological constnt to sipmplify.

Expanding the higgs field tree level higgs potential is like this.

We consider the corrections to this potential.

Now we take the gravitational flctuation from Minkowski backgroud.

Gravitational 1-loop effect

$$
\begin{aligned}
V_{loop} \; = \; & \frac{3}{64\pi^2}\left(-m^2+\lambda\phi^2\right)^2\left(\ln\frac{-m^2+\lambda\phi^2}{\Lambda^2}-\frac{3}{2}\right) \\
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& + \sum_{i=\pm}\frac{C_i^2}{64\pi^2}\left(\ln\frac{C_i}{\Lambda^2}-\frac{3}{2}\right)
\end{aligned}
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C_{\pm} = \frac{1}{2}\left\{V''_{tree}-\frac{16}{M_{Pl}^2}V_{tree}\pm\sqrt{\left(V''_{tree}+\frac{16}{M_{Pl}^2}V_{tree}\right)^2-\frac{128}{M_{Pl}^2}{V'_{tree}}^2}\right\}
$$

F. Loebbert and J. Plefka 2015

Condition of stability

Existance of higher terms : Φ^6 , Φ^8 ➡ ~~λ>0~~

Evaluate the effective potential

Then, This is 1-loop corrections to higgs potential.

This corrections was already calculated by Loebbert &Plefka this year

Second & third terms include gravitational effect , they are suppressed by Planck mass at electro-week scales.

Gravitational corrections make Φ to the 6 th & 8th potential. But these terms do not exist on Standared model .

Due to these terms we can not evaluate the vacuum stability with using the condition, lambda is positive.

So we studied the effective potential 's behavior at Planck scales.

Counter terms

Couplings of Φ^6 , Φ^8

These values are unkown

Set up

Φ^2 , Φ^4	Φ^6 , Φ^8
renormalizable	Non renormalizable

Φ^6 and Φ^8 terms depend on cut off

Counter terms

$$V_{counter} = A\,ln\frac{\Lambda^2}{\mu^2}\cdot\phi^2 + B\,ln\frac{\Lambda^2}{\mu^2}\cdot\phi^4$$

Annalyze the behavior

Effective potential

$$V_{eff} = V_{tree} + V_{loop} + V_{counter}$$

Higher power terms of Higgs field do not exist in SM.

We can not calculate the value of these terms's couplings

So we set these terms to zero in the bare action,, we consider quantum corrections tot their terms.

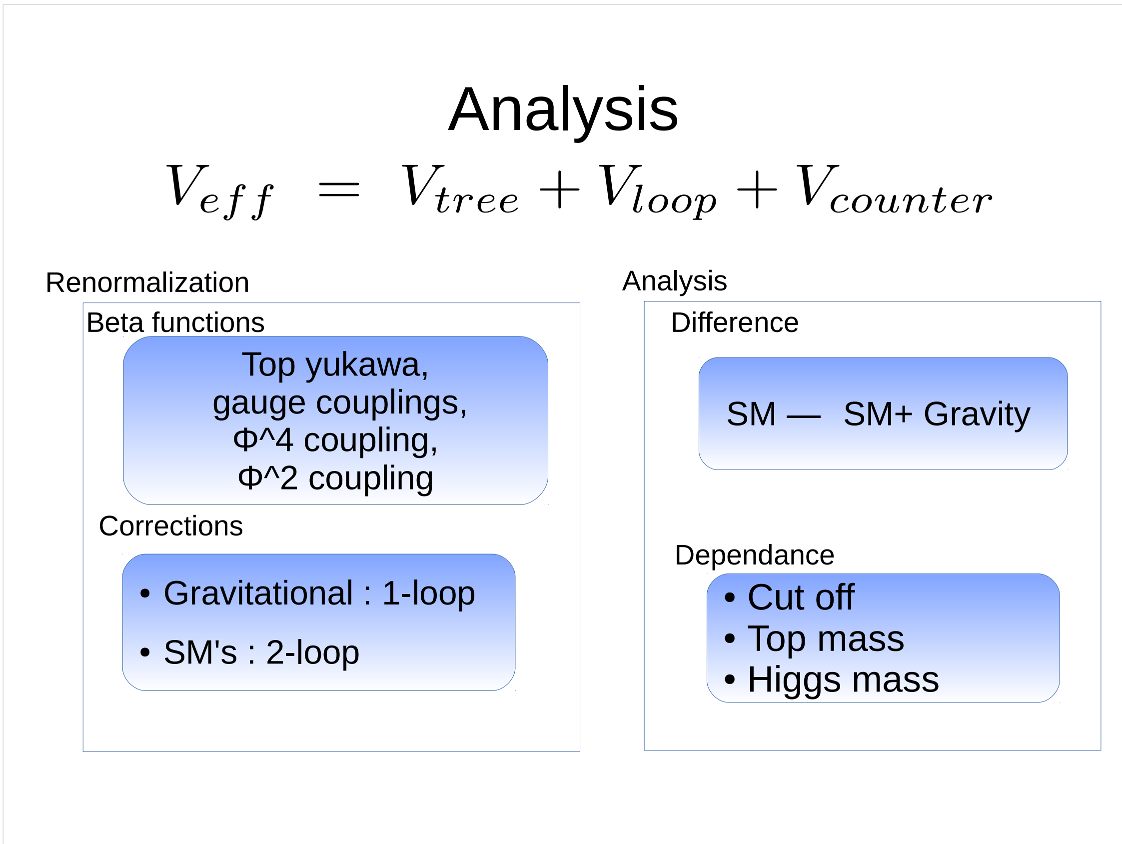
In this set up, we can get counter terms like this.

Then Φ to the 6th & 8th terms couplings depend on cut off.

Now we can calculate these value of couplings.

By these counter terms , we can get Higgs's effective potential like this.

So let'sanalyze the potential.



Now we set the higher terms couplings to be zero in the bare action.

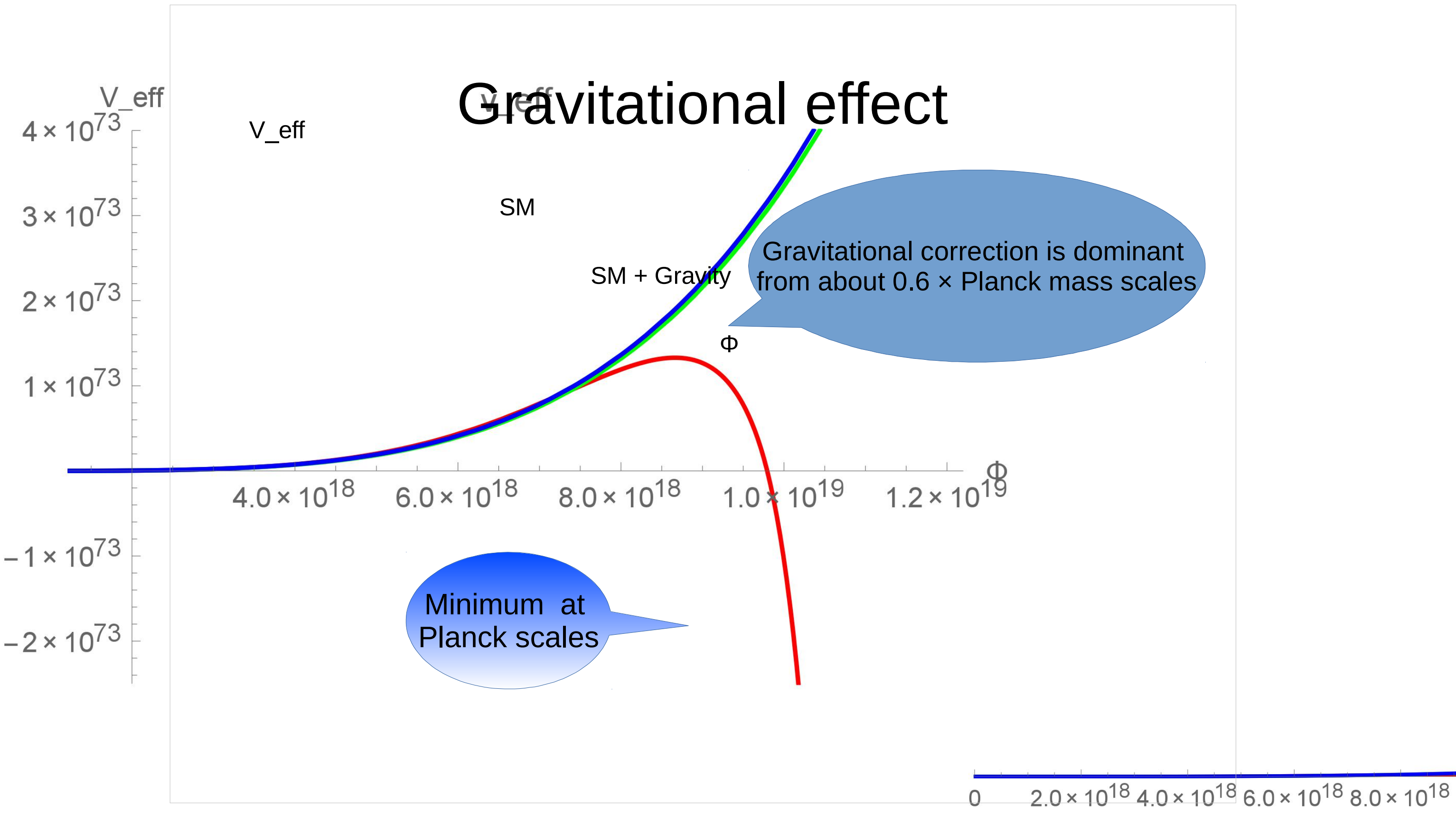
Hence we use the beta functions about only SM's couplings. like these, to compute the effective potential behavior.

Then we consider gravitaional 1-loop level corrections, and SM's 1-loop & 2-loop corections.

We analyze the effective potential about the following two points.

1. difference between SM and SM coupled to gravity.

2. dependance of cut off , top mass and Higgs mass.



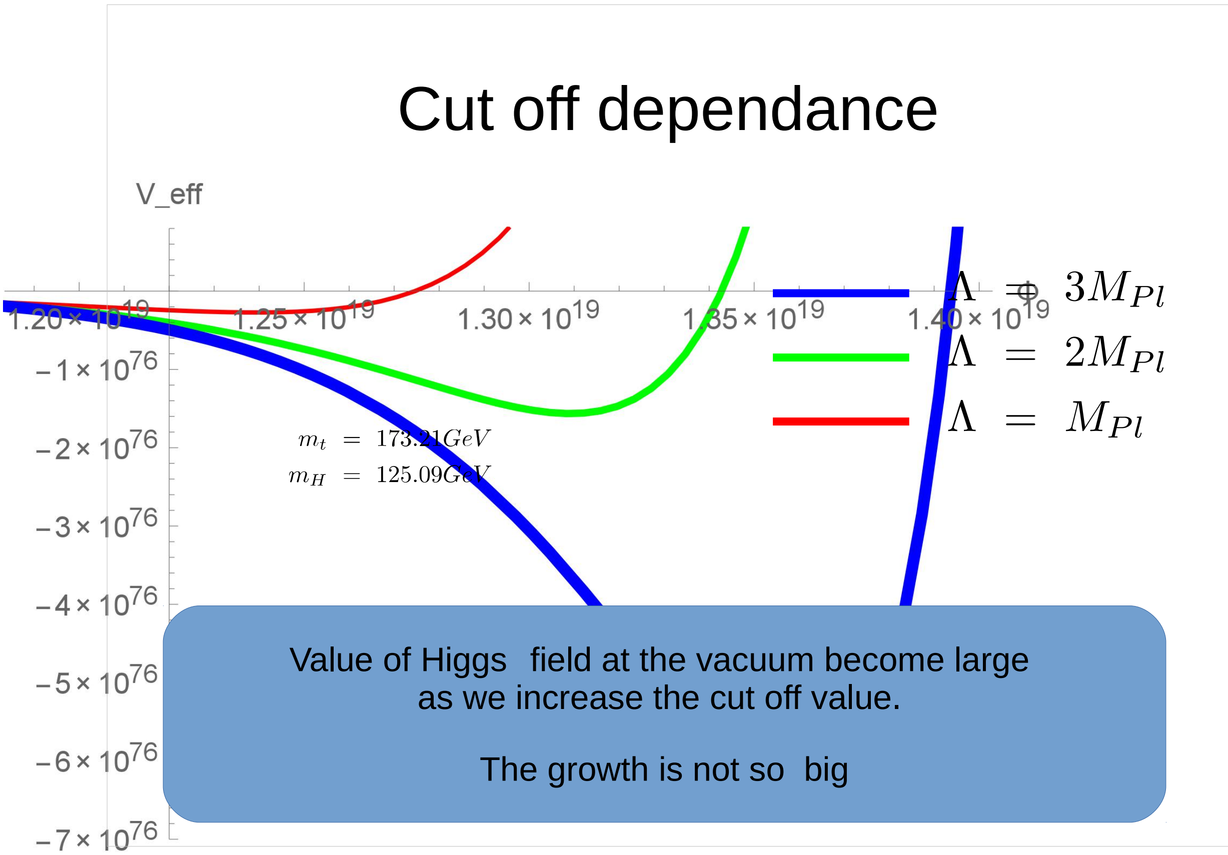
First I am going to talk about difference between SM and SM coupled to gravity

These are plot of effective potential.

Blue line shows SM's and Red line is SM with gravity.

Left plot show that Gravitational correction become to be effectie from about 0.6 × Planck mass scales

And right side plot show that the gravity correction make the minimum arround Planck scales.



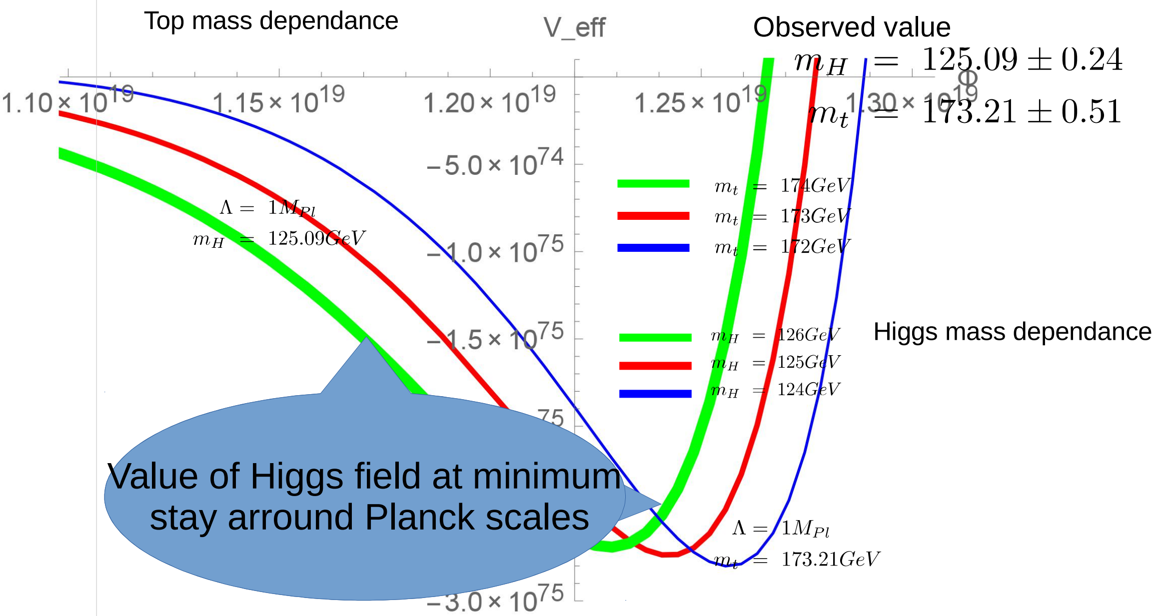
Next I am going to talk each dependances of the minimum.

This plot show the cut off dependance of effective potential.

The differences of the value of potential minimums are big.

But the value of Higgs field's at the minimum is not so changed.

Top& Higgs mass dependance



These are plot of top mass dependance & Higgs mass dependance.

Left figure is the plot of top dependance.
We set the conditions , cut off isM_{Pl} and Higgs mass is the observed value.

We can not find the big difference of the minimum between each lines.

These minimums are not so changed with in the error margins(or error range)

Summary

- We evaluate the quantum gravitational corrections to Higgs potential.
- Quantum Gravitational corrections become effective from about M_{Pl}/2.
- Gravitational corrections to Higgs potential make minimum at Planck scales.

Let me summarise.

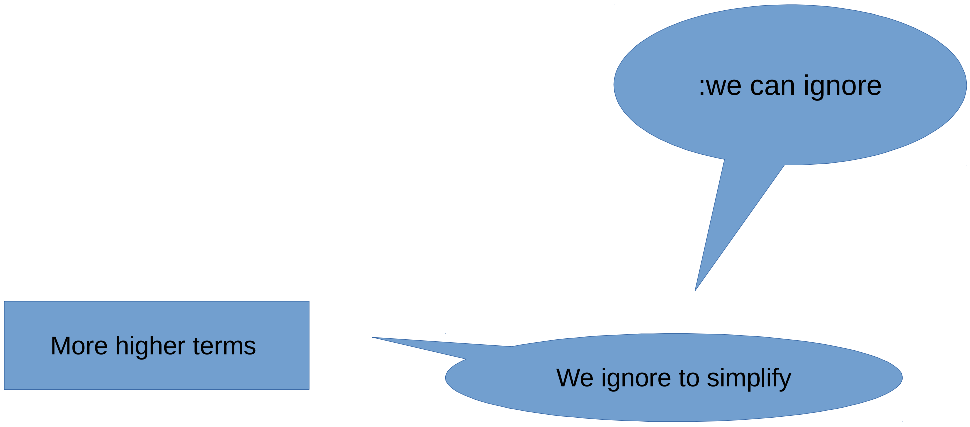
In this study, We evaluate the quantum gravitational corrections to Higgs potential.

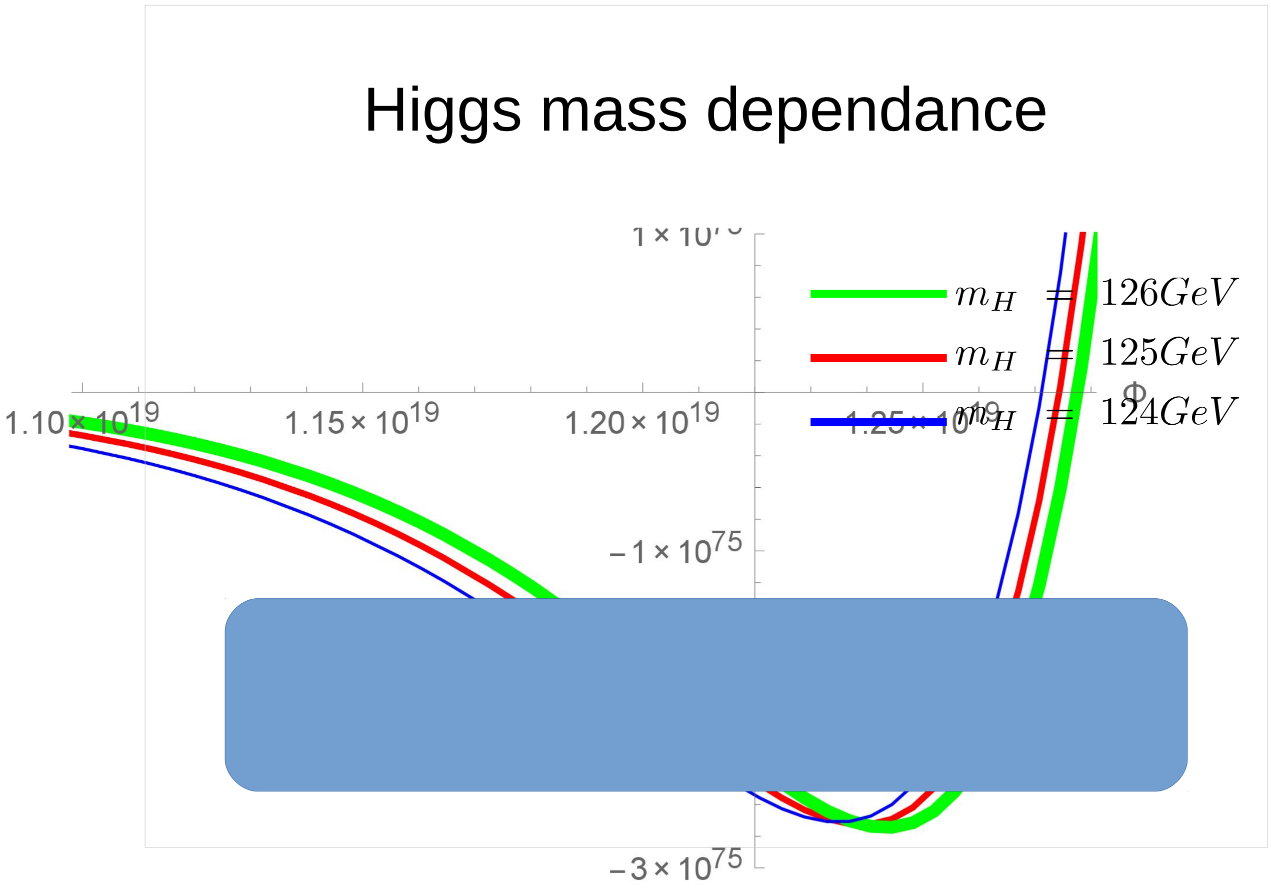
And we find that minimum of the Higgs potential is arround Planck scale.

I will stop here

Thank you

Higher terms




Higgs mass dependance
1 × 10^75
−1 × 10^75
−3 × 10^75
1.10 × 10^19
1.15 × 10^19
1.20 × 10^19
1.25 × 10^19
ϕ
$m_H = 126GeV$
$m_H = 125GeV$
$m_H = 124GeV$

Analysis

$$V_{eff} = V_{tree} + V_{loop} + V_{counter}$$

Difference

SM — SM+ Gravity

Observed value

$$m_H = 125.09 \pm 0.24$$
$$m_t = 173.21 \pm 0.51$$

Dependance

- Cut off
- Top mass
- Higgs mass

Scope of analysis

$$\Lambda = 1 \sim 3M_{Pl}$$
$$m_t = 173 \pm 1GeV$$
$$m_H = 125 \pm 1GeV$$

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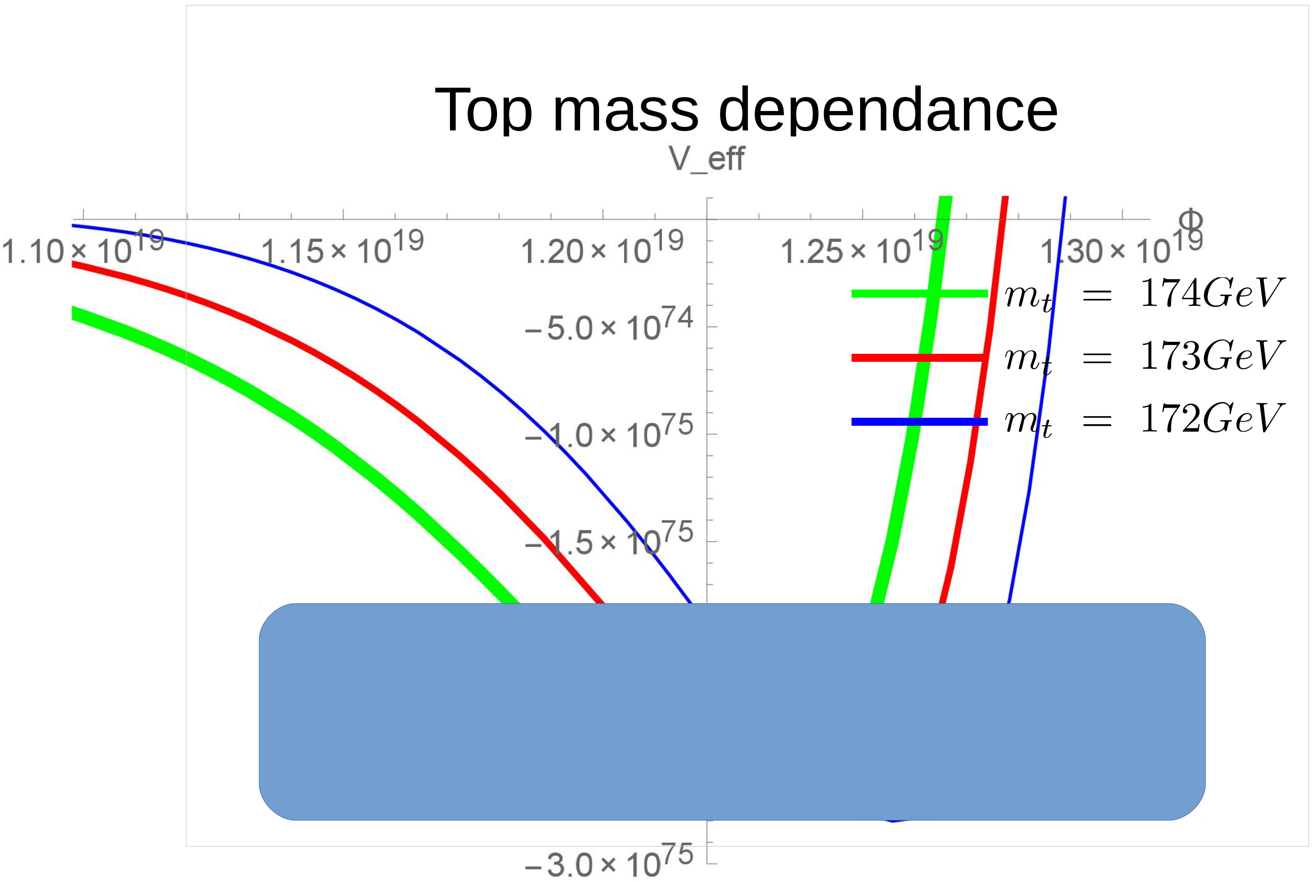
Top mass dependance
V_eff
Φ
1.10 × 10^19
1.15 × 10^19
1.20 × 10^19
1.25 × 10^19
1.30 × 10^19
−5.0 × 10^74
−1.0 × 10^75
−1.5 × 10^75
−3.0 × 10^75
$m_t = 174GeV$
$m_t = 173GeV$
$m_t = 172GeV$