Widening the Axion Field Range from Mixings

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Based on the work with
Gary Shiu and Wieland Staessens
1503.01015 [hep-th] (Phys.Rev.Lett. 115 (2015) 18, 181601), 1503.02965 [hep-th] (JHEP 1506 (2015) 026)

Introduction-Axions

- Axions: **CP-odd real scalars with continuous shift symmetry**; first introduced to solve strong CP problem

$$a(x) \to a(x) + \delta(x)$$

- Rich applications in particle physics and cosmology

Introduction-Axions

$$a(x) \to a(x) + \delta(x)$$

- **Continuous shift symmetry** $\longrightarrow$ Derivative couplings $\longrightarrow$ $\boxed{\frac{1}{f_a} da \wedge \star_4 J}$
  Possible light dark matter (DM) and inflaton?

- **Continuous shift symmetry is exact at most perturbatively.**

- Break continuous shift symmetry **down to discrete shift symmetry by non-perturbative** instantons

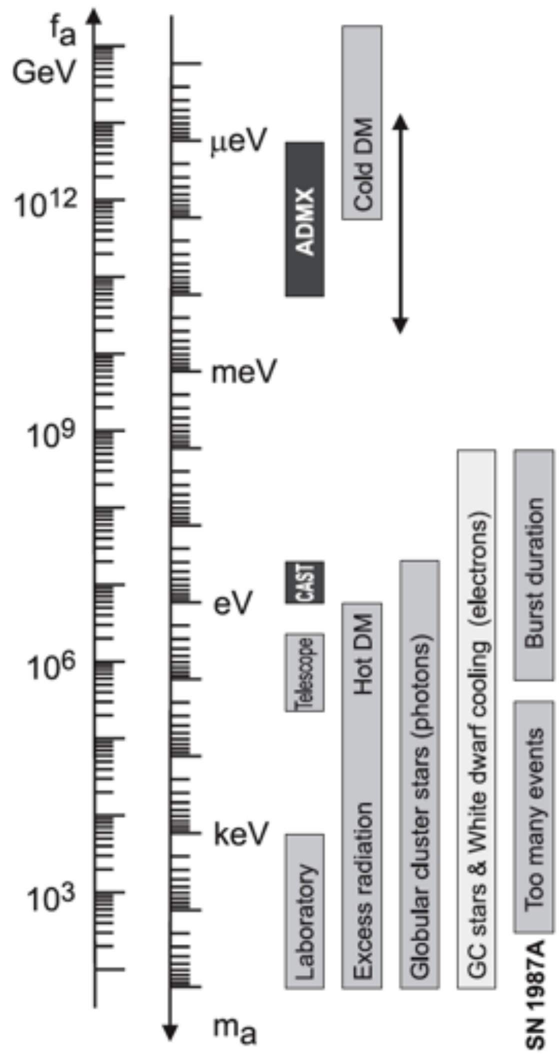
$$a(x) \to a(x) + 2\pi f_a$$

Axion decay constant: periodicity, field range

- Axion mass given by NP effects

$$m_a^2 = \frac{\partial^2 V_{eff}(a)}{\partial a^2} = \frac{\Lambda^4}{f_a^2}, \quad \Lambda = \text{NP scale}$$

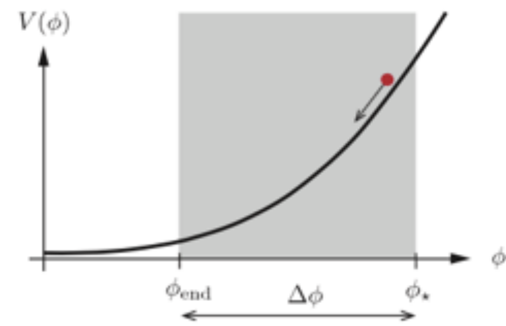
Introduction-Scenario 1: Axions as DM



$$10^9\,\mathrm{GeV} \le f_a^{DM} \le 10^{12}\,\mathrm{GeV}$$

(From PDG 2013)

Introduction-Scenario 2: Axions as Inflaton



- **Natural inflation**: periodic inflaton potential

$$V(a) = \Lambda^4[1 - \cos\frac{a}{f_a}]$$

- Slow roll:

$$\boxed{\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1,\ |\eta| \equiv M_P^2\left|\frac{V''}{V}\right| \ll 1 \Rightarrow f_a > M_P}$$

Large field inflation

Introduction-Stringy Axions

- Arise from dimensional reduction of higher forms (bulk fields)
- Shift symmetry as remnant of gauge symmetry of the forms in extra dimensions
- Constraint from string compactifications (no axion mixing):

$$f_a \lesssim \frac{g^2}{8\pi^2} M_P$$

Sub-Planckian!

P. Svrcek and E. Witten, JHEP **0606**, 051 (2006)
T. Banks, M. Dine, P. J. Fox and E. Gorbatov, JCAP **0306**, 001 (2003)

typically a little lower than string scale

Not in ADM or slow-roll inflation range

Introduction-Scenario 2: Axions as Inflaton

To have large field inflation:

- N-flation — Dimopoulos-Kachru-McGreevy-Wacker (2005)
- Aligned natural inflation (KNP) — Kim-Nilles-Peloso (2004), Choi-Kim-Yun (2014)
- Axion monodromy — Silverstein-Westphal (2008), McAllister-Silverstein-Westphal (2008), Kaloper-(Lawrence)-(Sorbo) (2008/11/14), Marchesano-Shiu-Uranga (2014), Franco-Galloni-Retolaza-Uranga (2014), McAllister-Silverstein-Westphal-Wrase (2014), Hebecker-Kraus-Witkowski (2014), Blumenhagen-Herschmann-Plauschinn (2014), Retolaza-Uranga-Westphal (2015)
- ?

Introduction-Motivation

- Widen the axion window without violating the string axion bound
- Apply both to field and string theories

Axion Mixings

- General lagrangian

Abelian

Non-Abelian

Fermion and/or GCS terms to ensure the gauge invariance

$$\boxed{\begin{aligned}\mathcal{S}^{\text{eff}} = -\int \Bigg[&\sum_{\alpha,\,\beta=1}^{M} f_{\alpha\beta} F^{\alpha} \wedge \star_4 F^{\beta} + \sum_{A=1}^{P} \frac{1}{g_A^2} \mathrm{Tr}\,(G^A \wedge \star_4 G^A) - \frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (da^i - \sum_{\alpha=1}^{M} k_\alpha^i A^\alpha) \wedge \star_4 (da^j - \sum_{\beta=1}^{M} k_\beta^j A^\beta) \\ &- \frac{1}{8\pi^2} \sum_{A=1}^{P} \left(\sum_{i=1}^{N} r_{iA} a^i \right) \mathrm{Tr}(G^A \wedge G^A) - \frac{1}{8\pi^2} \sum_{\alpha,\beta=1}^{M} \left(\sum_{i=1}^{N} s_{i\alpha\beta} a^i \right) F^\alpha \wedge F^\beta + \ldots \Bigg] .\end{aligned}}$$

Winding number, anomalous coefficient etc

"Axion charge"

- Basis and normalization $k_\alpha^i,\ r_{iA},\ s_{i\alpha\beta} \in \mathcal{Z}$

Compactness of U(1)'s

$$U(1)^\alpha :\ A^\alpha \to A^\alpha + d\eta^\alpha,\ a^i \to a^i + k_\alpha^i \eta^\alpha$$

Axion Mixings

determined by the vev of saxions

$$\mathcal{S}^{\text{eff}} = -\int \Bigg[\sum_{\alpha,\,\beta=1}^{M} f_{\alpha\beta} F^\alpha \wedge \star_4 F^\beta + \sum_{A=1}^{P} \frac{1}{g_A^2} \mathrm{Tr}\,(G^A \wedge \star_4 G^A) - \frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (da^i - \sum_{\alpha=1}^{M} k^i_\alpha A^\alpha) \wedge \star_4 (da^j - \sum_{\beta=1}^{M} k^j_\beta A^\beta) - \frac{1}{8\pi^2} \sum_{A=1}^{P} \left(\sum_{i=1}^{N} r_{iA} a^i \right) \mathrm{Tr}(G^A \wedge G^A) - \frac{1}{8\pi^2} \sum_{\alpha,\beta=1}^{M} \left(\sum_{i=1}^{N} s_{i\alpha\beta} a^i \right) F^\alpha \wedge F^\beta + \ldots \Bigg].$$

- **3 types of axion mixings:**

- Mixing due to ***non-diagonal metric*** (referred as **kinetic mixing or metric mixing**)

non-vanishing charges

- Mixing due to ***Stueckelberg couplings*** (referred as **Stueckelberg mixing**)
- Mixing due to **mismatch between kinetic eigenstates and mass eigenstates**

Axion-Mixings (minimal)

$$S_{axion}^{N=2} = \int \left[-\frac{1}{2} \sum_{i,j=1}^{2} \mathcal{G}_{ij} \left(da^i - k^i A \right) \wedge \star_4 \left(da^j - k^j A \right) + \frac{1}{8\pi^2} \left(r_1 a^1 + r_2 a^2 \right) \mathrm{Tr}\, G \wedge G + \ldots \right]$$

- Minimal setup: **2 axions**, **1 Stueckelberg U(1)** and **1 non-Abelian gauge group**

Axion Mixings (minimal)

W/O Stueckelberg coupling $$\mathcal{S}^{\rm kin}_{axion} = -\int \frac{1}{2}\sum_{i,j=1}^{2} \mathcal{G}_{ij}(\sigma)\, \mathrm{d}a^i \wedge \star_4 \mathrm{d}a^j ,$$

flat direction whose continuous shift symmetry is not broken has no potential

axionic couplings scale inversely with the axion decay constant.

can't have super-Planckian excursion

have to invoke additional physical effects

(1) *Monodromy effects.* $V(\xi) \sim \xi^p$

Through torsional monodromy effects (p=2), or flux induced monodromies (p>2)

(2) *Alignment effects.*

Add a second non-Abelian gauge group anomalously coupling to both axions

(3) *Abelian $U(1)$ gauge symmetry.* **Will study this in detail**

Axion Mixings (minimal)

In the presence of Stueckelberg coupling

Kinetic eigenstates:

- **Axion eaten by U(1)**: part of the massive U(1) in appropriate gauge
- **Uneaten axion**: obtain **effective potential for inflation** by integrating out massive U(1), chiral fermions and non-Abelian gauge field

$$V_{\rm eff}(\xi) = \Lambda^4 \left[1 - \cos\left(\frac{\xi}{f_\xi}\right)\right]$$

Single field potential for the remaining axion

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-} M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+ k^+ r_2 + \lambda_- k^- r_1\right) + \sin\frac{\theta}{2}\left(\lambda_- k^- r_2 - \lambda_+ k^+ r_1\right)}$$

mixing angle: amount of metric mixing

Axion Mixings (minimal)

Large field inflation

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2 + \lambda_-k^-r_1\right) + \sin\frac{\theta}{2}\left(\lambda_-k^-r_2 - \lambda_+k^+r_1\right)}$$

- Explore axion field range - intermediate kinetic mixing

$$\varepsilon^2 \equiv \mathcal{G}_{22}/\mathcal{G}_{11}$$

White region $f_\xi > 10^2\sqrt{\mathcal{G}_{11}}$

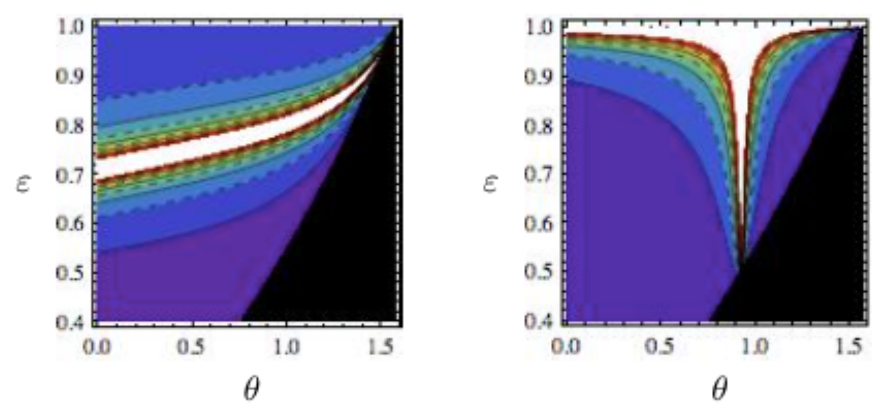


FIG. 1. Contour plots of decay constant $f_\xi(\theta,\varepsilon)$ for $2r_1 = 2r_2 = 2k^1 = k^2$ (left) and $r_1 = 2r_2 = k^1 = 2k^2$ (right). The f_ξ-values range from small (purple) to large (red) following the rainbow contour colors. Unphysical regions with complex f_ξ are located in the black band.

Axion Mixings (minimal)

Other axion large field inflation scenarios:

Both tied to number of DOF

- **N-flation**: S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, JCAP **0808**, 003 (2008)

$f_{eff} \propto \sqrt{N}$, N = number of axions = number of non-Abelian gauge instaontons

N=2: some anomalous coefficients at order of 100

- **Aligned natural inflation**: J. E. Kim, H. P. Nilles and M. Peloso, JCAP **0501**, 005 (2005)

$f_{eff} \propto \sqrt{N!}n^N$, N>2 version K. Choi, H. Kim and S. Yun, Phys. Rev. D **90**, no. 2, 023545 (2014)

N = number of axions = number of non-Abelian gauge instantons,

$n \in \mathcal{Z}$ anomalous coefficients

- Planck mass renormalization $\delta M_{Pl}^2 \sim N$

Spoil the slow roll condition?

Axion Mixings (minimal)

Our approach to get a super-Planckian inflation:

- Axion field range enhancement **not tied to the number of DOF**
- Relying on tuning **continuous parameters** in moduli space, not much on discrete parameters
- **Minimal (fewer DOF) setup works, which has less severe Planck mass renormalization issue.**

Axion Mixings (ADM)

Minimal setup

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2+\lambda_-k^-r_1\right)+\sin\frac{\theta}{2}\left(\lambda_-k^-r_2-\lambda_+k^+r_1\right)}$$

- **To lower axion decay constants**: by tuning continuous parameters and choosing appropriate discrete parameters

$\varepsilon^2 \equiv \mathcal{G}_{22}/\mathcal{G}_{11}$

$\varrho^2 \equiv \mathcal{G}_{12}/\mathcal{G}_{11}$

e.g. assuming $\varepsilon = 1,\ \theta = \pi,\ r_1 = -r_2,\ k^1 = k^2$

$$f_\xi = \frac{\sqrt{\mathcal{G}_{11}^2-\mathcal{G}_{12}^2}}{\sqrt{2}r_2\sqrt{\mathcal{G}_{11}+\mathcal{G}_{12}}} = \frac{\sqrt{\mathcal{G}_{11}}\sqrt{1-\varrho^2}}{\sqrt{2}r_2}$$

$r_2 \sim \mathcal{O}(1-10),\ \sqrt{\mathcal{G}_{11}} \sim \mathcal{O}(10^{15}-10^{17})\,\mathrm{GeV},$ large mixing $1-\rho^2 \sim \mathcal{O}(10^{-4}-10^{-8})$

Not require the intrinsic axion field range to be too small

Non-minmal setup

For large N, eigenvalue repulsion

String Theory Embedding

- Approaching Planck scale: **physics is sensitive to Planck scale**
- Need a full theory description $\rightarrow$ String Theory

String Theory Embedding

- **Closed string axions (Ramond-Ramond axions)**: dimensional reduction of p-forms $a^i \equiv \frac{1}{2\pi} \int_{\gamma_i} \mathcal{C}_p$

- Number of RR axions: Hodge numbers

String Theory Embedding

- **Axion metric** $\mathcal{G}_{ij}$: moduli field dependent (stabilization of the saxion, e.g. volume of the internal cycle wrapped by p-forms)

- **U(1)**: Abelian factor of **world volume gauge group U(N)** for a stack of N D-branes

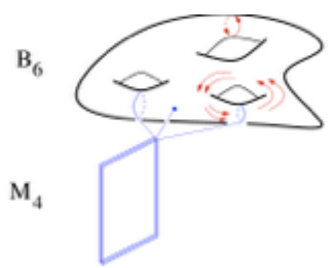
- **Gauge Dp-branes**: extending along Minkowski space and wrapping internal (p-3)-cycles

- **Wrapping numbers** $\rightarrow$ Discrete parameters k^i_α $\quad r_{i\alpha\beta}$

String Theory Embedding

- Instantons: **Gauge instantons** or **stringy instantons**

- **Gauge instanton**: on world volume of Dp-brane extending Minkowski space and wrapping internal (p-3)-cycles



(Graph from 1003.4867)

$$e^{-S_{gauge}} = e^{-|I_n|\left(\frac{8\pi^2}{g_{YM}^2}+i\,\theta\right)}$$

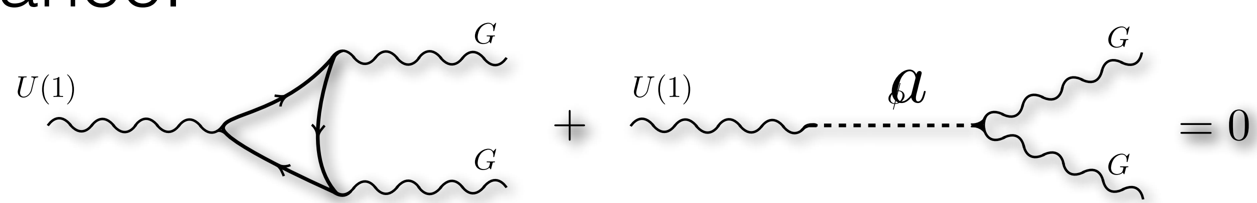
- **Stringy instanton**: e.g. E(p-1)-instanton (D(p-1)-brane wrapping internal p-cycles, pointlike in 4d).

$$e^{-S_{E_{q-1}}} = e^{-\frac{2\pi}{\ell_s^q}\left(\frac{1}{g_s}\mathrm{Vol}(\gamma_i)+i\int_{\gamma_i} C_q\right)} = e^{-\frac{2\pi}{\ell_s^q}\frac{1}{g_s}\mathrm{Vol}(\gamma_i)-i\,a^i}$$

- **Instantons contribute only when fermonic zero modes are saturated**.

String Theory Embedding

- Introducing orientifold planes
- Consistency: tadpole cancellation
- Chiral spectrum: bi-fundamentals at intersections
- Green-Schwarz (GS) mechanism to ensure gauge invariance.

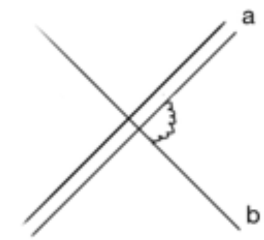


String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

2 stacks of D6's wrapping ***non-factorisable*** 3-cycles

Internal space $T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$



Other explicit D-brane models to realize super-Planckian excursion

Factorizable D6-branes in Type IIA on Toroidal Orientifolds

Not all the RR tadpoles and the related gauge anomalies vanish.
Need to be remediated.

D7-branes in Type IIB on Swiss-Cheese Calabi-Yau's

Consistency (tadpole condition) to be checked in the future

String Theory Embedding

Other related questions

- Moduli stabilization **many attempts**

recent **Blumenhagen-Fuchs-Herschmann: 1510.04058 [hep-th]**

- Weak gravity conjecture (WGC) Arkhani-Hamed-Motl-Nicolis-Vafa (2006)

Brown-Conttrell-Shiu-Soler: 1503.04783, 1504.00659 [hep-th] **Strong -> not compatible w/ LFI**

- Higher instanton corrections: not ruin the enhancement for the axion decay constant; but add modulations on the potential

Kappa-Nilles-Winkler:1511.05560 [hep-th]

+ all stringy instanton corrections: super-Planckian excursion compatible with WGC (mild)

Choi-Kim: 1511.07201 [hep-th]

Not all instantons participate in alignment; some generate small modulations instead, consistent with WGC (mild).

Conclusion

Kinetic and Stueckelberg mixings can enlarge axion field range

- Applies to both field theory and string theory models with limited intrinsic axion field ranges.

- Axion field range enhancement is mainly through **tuning continuous parameters** (with discrete parameters properly chosen).

- **To lower the axion decay constant (for ADM)**: **no large compact cycles** needed (alleviate the requirement for intermediate string scale). **Allow high fundamental string scale** -> **new possibilities to detect string axions** through astrophysical, cosmological and laboratory ways

- **To increase the axion decay constant (for inflation)**: **not require large DOF** (-> minimal setup) and mainly depends on tuning continuous parameters.

- Model-dependent **higher instanton corrections**: **deviation from a cosine potential -> measurable effect on the inflationary perturbation spectrum**. Quantifying such deviation needs understanding of UV completion of inflation and moduli stabilization.

Thank you!

Backup

Stueckelberg Mechanism

4d QFT with a massive U(1) and an axion

$$\mathcal{L}_0 = -\frac{1}{4g^2}(F_{\mu\nu})^2 - \frac{1}{2}(cA_\mu + \partial_\mu a)^2$$

Add a "vanishing" term $\frac{1}{6} a \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma}$ with $H_{(3)} = \mathrm{d}B_{(2)}$

E.O.M for a

$$\partial_\mu (cA^\mu + \partial^\mu a) + \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma} = 0.$$

Up to a total derivative

$$H_{\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}(cA^\mu + \partial^\mu a).$$

$$\int_{\mathrm{D6}_a} C_{(5)} \wedge F_a;$$

dimensional reduction to 4d

$$\sum_i r_i^a \int_{M_4} B^i_{(2)} \wedge F_a$$

e.g.

Eliminating the axion field

$$\mathcal{L}_0 = -\frac{1}{4g^2}(F_{\mu\nu})^2 - \frac{1}{2}(-\frac{1}{6}\epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma})^2 - \frac{1}{6}\epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}(-cA_\mu - \frac{1}{6}\epsilon_{\mu\alpha\beta\gamma} H^{\alpha\beta\gamma})$$
$$= -\frac{1}{4g^2}(F_{\mu\nu})^2 - \frac{1}{12}(H_{\mu\nu\rho})^2 + \frac{c}{4}\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma}.$$

terms read from string compactifications

Eta Problem

- Eta parameter $|\eta| \equiv M_{\rm pl}^2 \frac{|V''|}{V}$
- Quantum corrections tend to drive the scalar mass to the cutoff $\Delta m^2 \sim \Lambda^2$
- Consistency of EFT: $\Lambda > H$
- -> Large enormalization of eta parameter $\Delta\eta \sim \frac{\Lambda^2}{H^2} \gtrsim 1$
- Snow-roll unnatural
- In fact, eta parameter is sensitive to dim-6 operator correction $\mathcal{O}_6 = cV_l(\phi)\frac{\phi^2}{\Lambda^2}$ $\Delta\eta \approx 2c\left(\frac{M_{\rm pl}}{\Lambda}\right)^2$

RR Tadpole Cancellation

Emission of a closed string out of a vacuum -> a **tadpole**

10d Poincare invariance: the source fields can be graviton and dilaton in NS-NS sector, and 10-form in RR sector C_{10}

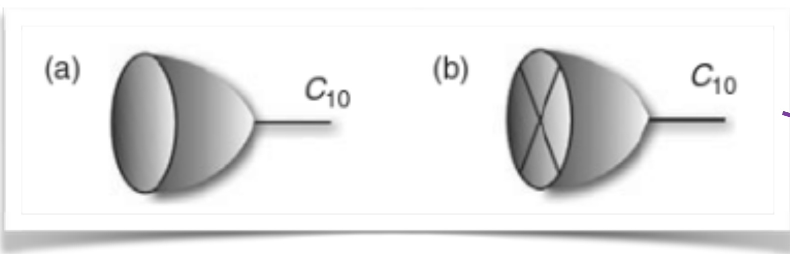
RR 10-form is not propagating (its field strength is 11d) in 10d -> **can't appear in physical spectrum of spacetime particles**

But it appears in 10d action as a source term

$$S_{C_{10}} = N\, Q_{\text{disk}} \int_{M_{10}} C_{10}$$

RR charge

$$N = 0$$



Need cancellation

Bonus: RR tadpole cancellation leads to mixed gauge anomaly cancellation

Chiral Spectrum at D-brane Intersections in Type IIA

- Orientifold planes O6 are introduced to ensure RR tadpole (e.g. emission of a closed string out of vacuum, reflecting uncancelled gravitational/ gauge anomalies) cancellation. O6 has -4 units of D6-brane charge.

- Branes are mapped to image branes (denoted by prime) under orientifold action.

- For intersecting D-brane stacks a and b, with brane numbers N_a and N_b, open strings stretching between 2 stacks arise as bi-fundamentals.

Spectrum

- $aa + a'a'$: Contains $U(N_a)$ gauge bosons (plus possible additional adjoint fields).
- $ab + ba + b'a' + a'b'$: Gives I_{ab} chiral fermions in the representation $(\mathbf{N_a}, \overline{\mathbf{N}}_\mathbf{b})$, plus light (possibly massless) scalars.
- $ab' + b'a + ba' + a'b$: Contains $I_{ab'}$ chiral fermions in the representation $(\mathbf{N_a}, \mathbf{N_b})$, plus light (possibly massless) scalars.
- $aa' + a'a$: Contains $n_{\text{sym},a}$ 4d chiral fermions in the representation $\square\!\square_a$ and $n_{\text{asym},a}$ in the $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$, with $\quad n_{\text{sym},a} = \frac{1}{2}(I_{aa'} - I_{a,\text{O6}}), \quad n_{\text{asym},a} = \frac{1}{2}(I_{aa'} + I_{a,\text{O6}})$

Factorisable Tori

$$\boxed{T^2 \times T^2 \times T^2}$$

stack A of D-branes wraps the i-th torus $(T^2)^i$ $(i = 1, 2 \text{ or } 3)$, with wrapping numbers (n^i_A, m^i_A)

$[a_i]$ and $[b_i]$ be the even and odd 1-cycles on $(T^2)^i$ with respect to the antiholomorphic involutions $y^i \to -y^i$

basis of even and odd 3-cycles

$$\begin{aligned}
[\alpha_0] &= [a_1][a_2][a_3], \quad [\beta_0] = [b_1][b_2][b_3], \\
[\alpha_1] &= [a_1][b_2][b_3], \quad [\beta_1] = [b_1][a_2][a_3], \\
[\alpha_2] &= [b_1][a_2][b_3], \quad [\beta_2] = [a_1][b_2][a_3], \\
[\alpha_3] &= [b_1][b_2][a_3], \quad [\beta_3] = [a_1][a_2][b_3],
\end{aligned}$$

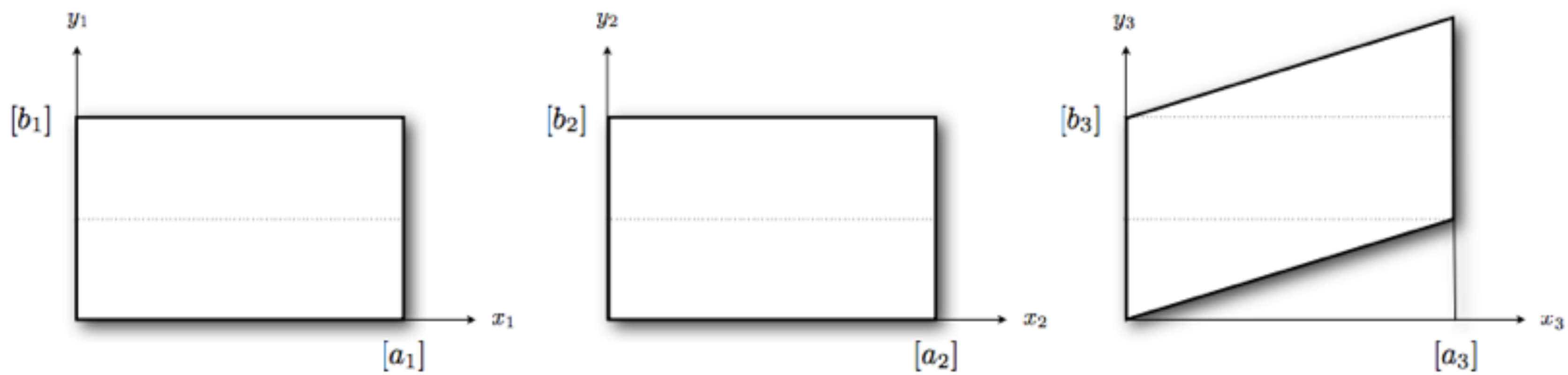
$$\begin{aligned}
[\alpha_i] \cdot [\beta_j] &= \delta_{ij} = -[\beta_j] \cdot [\alpha_i], \\
[\alpha_i] \cdot [\alpha_j] &= [\beta_i] \cdot [\beta_j] = 0.
\end{aligned}$$

3-cycles $[\pi_A]$, can be expanded in terms of the basis

$$[\pi_A] = S_A{}^i [\alpha_i] + R_A{}^i [\beta_i].$$

$$\begin{aligned}
S_A{}^0 &= n^1_A n^2_A n^3_A, \quad R_A{}^0 = m^1_A m^2_A m^3_A, \\
S_A{}^1 &= n^1_A m^2_A m^3_A, \quad R_A{}^1 = m^1_A n^2_A n^3_A, \\
S_A{}^2 &= m^1_A n^2_A m^3_A, \quad R_A{}^2 = n^1_A m^2_A n^3_A, \\
S_A{}^3 &= m^1_A m^2_A n^3_A, \quad R_A{}^3 = n^1_A n^2_A m^3_A.
\end{aligned}$$

Rectangular and Tilted Tori



Notice that $[a_3]$ does not represent a closed

one-cycle on T_3^2, only linear combinations such as $2[a_3]$ or $[a_3] + \frac{1}{2}[b_3]$ do.

Half integer wrapping number

$$y^i \to -y^i$$

$$(n_A^i, m_A^i) \to (n_A^i, -m_A^i)$$

$$m_A^i = \tilde{m}_A^i + \frac{n_A^i}{2} \quad \text{(tilted)}$$

$\tilde{m}_A^i$ and n_A^i being integers

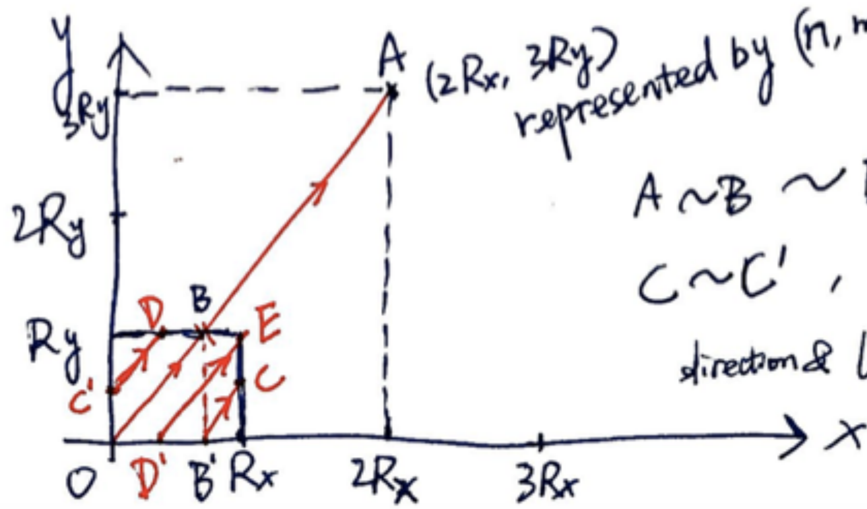
Rectangular and Tilted Tori

$\gcd(n,m) = 1$ $\qquad T^2$ $\qquad \boxed{\text{Orientifold action : } y \to -y}$

(n,m) represents closed line (oriented) on T^2.

e.g. $(n,m) = (2,3)$ w/ T^2 defined by $z \sim z + R_x$

$z \sim z + iR_y$



$(2R_x, 3R_y)$ represented by $(n,m) = (2,3)$

$A \sim B \sim B'$,

$C \sim C'$, $D \sim D'$

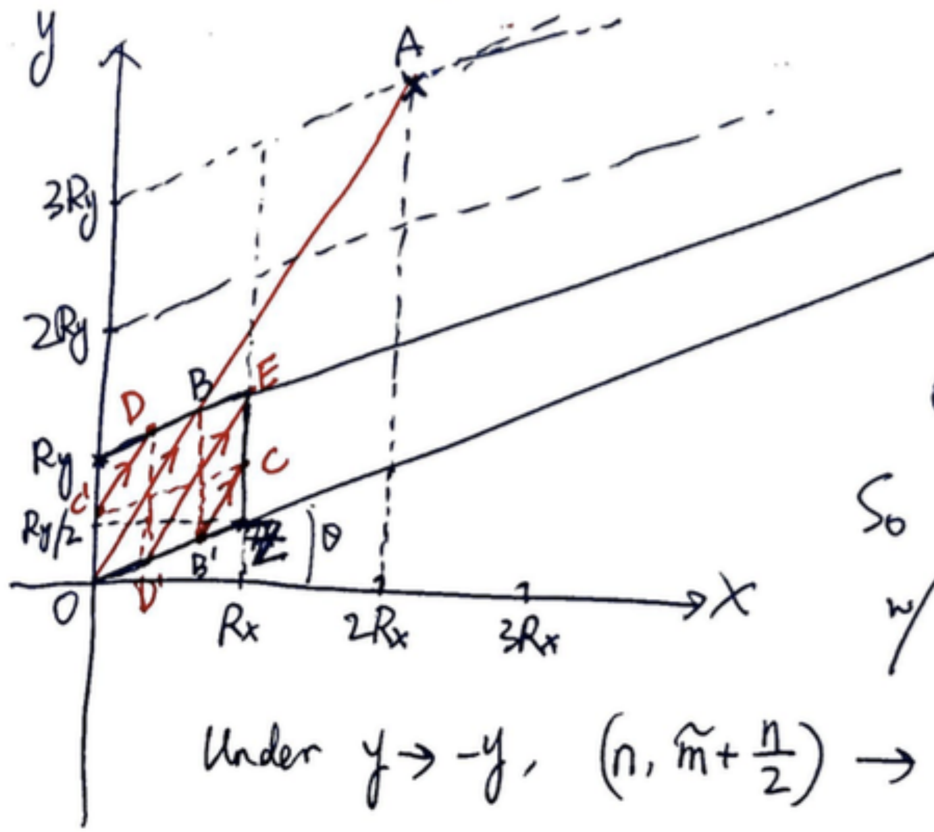
direction & length $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{B'C} + \overrightarrow{C'D} + \overrightarrow{D'E}$

Rectangular and Tilted Tori

e.g. 2 : T^2 defined by $z \sim z + iR_y$

$z \sim z + R_x + i\frac{R_y}{2}$

Wrap $\underset{\downarrow \atop n}{2}$ units along $\overrightarrow{O\mathcal{Z}}$, $\underset{\downarrow \atop \tilde{m}}{3}$ units along y-axis



~~A~~

Coordinates for A is

$nR_x + i(\tilde{m}R_y + nR_x \tan\theta)$

$= nR_x + i(\tilde{m}R_y + \frac{n}{2}R_y)$

$(n = 2,\ \tilde{m} = 3$ here$)$

So $(n, \tilde{m} + \frac{n}{2})$ represents A,

w/ $n, \tilde{m} \in \mathbb{Z}$

Under $y \to -y$, $(n, \tilde{m} + \frac{n}{2}) \to (n, -\tilde{m} - \frac{n}{2})$

Axion Mixings (minimal)

W/O Stueckelberg coupling

$$\mathcal{S}^{\text{kin}}_{axion} = -\int \frac{1}{2} \sum_{i,j=1}^{2} \mathcal{G}_{ij}(\sigma)\, \mathrm{d}a^i \wedge \star_4 \mathrm{d}a^j\,,$$

Metric

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \mathcal{G}_{12} & \mathcal{G}_{22} \end{pmatrix}, \qquad \text{with } \mathcal{G}_{11}, \mathcal{G}_{12}, \mathcal{G}_{22} \in \mathbb{R}\backslash\{0\}.$$

SO(2) rotation diagonalizing the metric:

$$\begin{pmatrix} a^- \\ a^+ \end{pmatrix} = \begin{pmatrix} \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} a^1 \\ a^2 \end{pmatrix}$$

$$\cos\theta = \frac{\mathcal{G}_{11} - \mathcal{G}_{22}}{\sqrt{4\mathcal{G}_{12}^2 + (\mathcal{G}_{11} - \mathcal{G}_{22})^2}}, \qquad \sin\theta = \frac{2\mathcal{G}_{12}}{\sqrt{4\mathcal{G}_{12}^2 + (\mathcal{G}_{11} - \mathcal{G}_{22})^2}}, \qquad \text{with } 0 \le \theta < 2\pi.$$

Rescale the axions s.t. the anomalous coupling is in a purely topological term:

$$\tilde{a}^- \equiv \left(r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2} \right) a^-, \qquad \tilde{a}^+ \equiv \left(r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2} \right) a^+ \qquad \hat{a}^+ \equiv f_{\tilde{a}^+}\, \tilde{a}^+, \qquad \hat{a}^- \equiv f_{\tilde{a}^-}\, \tilde{a}^+$$

$$V^{\text{eff}}_{axion}(\hat{a}^-, \hat{a}^+) = \Lambda^4 \left[1 - \cos\left(\frac{\hat{a}^-}{f_{\tilde{a}^-}} + \frac{\hat{a}^+}{f_{\tilde{a}^+}} \right) \right] \qquad f_{\tilde{a}^-} = \frac{\sqrt{\lambda_-}}{|r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2}|}, \qquad f_{\tilde{a}^+} = \frac{\sqrt{\boxed{\lambda_+}}}{|r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}|}.$$

Eigenvalues of the metric

Axion Mixings (minimal)

W/O Stueckelberg coupling

$$\mathcal{S}_{axion} = -\int \left[\frac{1}{2} d\hat{a}^- \wedge \star_4 d\hat{a}^- + \frac{1}{2} d\hat{a}^+ \wedge \star_4 d\hat{a}^+ + V^{\rm eff}_{axion}(\hat{a}^-, \hat{a}^+) \star_4 \mathbf{1}\right].$$

$$V^{\rm eff}_{axion}(\hat{a}^-, \hat{a}^+) = \Lambda^4 \left[1 - \cos\left(\frac{\hat{a}^-}{f_{\tilde{a}^-}} + \frac{\hat{a}^+}{f_{\tilde{a}^+}}\right)\right] \qquad f_{\tilde{a}^-} = \frac{\sqrt{\lambda_-}}{|r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2}|}, \qquad f_{\tilde{a}^+} = \frac{\sqrt{\lambda_+}}{|r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}|}.$$

Trans-Planckian excursion possible?

Need to find eigenbasis of the mass square matrix

$$\begin{pmatrix} \boxed{\xi} \\ \zeta \end{pmatrix} = \frac{1}{\sqrt{f^2_{\tilde{a}^+} + f^2_{\tilde{a}^-}}} \begin{pmatrix} f_{\tilde{a}^+} & -f_{\tilde{a}^-} \\ f_{\tilde{a}^-} & f_{\tilde{a}^+} \end{pmatrix} \begin{pmatrix} \hat{a}^+ \\ \hat{a}^- \end{pmatrix} \qquad \mathcal{S}_{axion} = -\int \left[\frac{1}{2} d\xi \wedge \star_4 d\xi + \frac{1}{2} d\zeta \wedge \star_4 d\zeta + V^{\rm eff}_{axion}(\zeta) \star_4 \mathbf{1}\right]$$

$$V^{\rm eff}_{axion}(\zeta) = \Lambda^4 \left[1 - \cos\left(\frac{\sqrt{f^2_{\tilde{a}^+} + f^2_{\tilde{a}^-}}}{f_{\tilde{a}^+} f_{\tilde{a}^-}} \zeta\right)\right] \qquad f_{\rm eff} = \frac{f_{\tilde{a}^+} f_{\tilde{a}^-}}{\sqrt{f^2_{\tilde{a}^+} + f^2_{\tilde{a}^-}}}$$

can't be super-Planckian!

flat direction whose continuous shift symmetry is not broken

axionic couplings scale inversely with the axion decay constant.

Axion Mixings (minimal)

W/O Stueckelberg coupling

$$V^{\rm eff}_{axion}(\zeta) = \Lambda^4 \left[1 - \cos\left(\frac{\sqrt{f^2_{\tilde{a}^+} + f^2_{\tilde{a}^-}}}{f_{\tilde{a}^+} f_{\tilde{a}^-}}\zeta\right)\right] \qquad \begin{pmatrix} \xi \\ \zeta \end{pmatrix} = \frac{1}{\sqrt{f^2_{\tilde{a}^+} + f^2_{\tilde{a}^-}}} \begin{pmatrix} f_{\tilde{a}^+} & -f_{\tilde{a}^-} \\ f_{\tilde{a}^-} & f_{\tilde{a}^+} \end{pmatrix} \begin{pmatrix} \hat{a}^+ \\ \hat{a}^- \end{pmatrix}$$

can't have super-Planckian excursion

if we interpret the axion ξ as the inflaton candidate

have to invoke additional physical effects

(1) *Monodromy effects.* $\quad V(\xi) \sim \xi^p$

Through torsional monodromy effects (p=2), or flux induced monodromies (p>2)

(2) *Alignment effects.*

Add a second non-Abelian gauge group anomalously coupling to both axions

(3) *Abelian $U(1)$ gauge symmetry.* **Will study this in detail**

Axion Mixings (minimal)

How to ensure gauge invariance?

$$\mathcal{S}_{sub} = \int \left[-\frac{f_{\tilde{a}^2}^2}{2} \left(d\tilde{a}^2 - \tilde{k}^2 A \right) \wedge \star_4 \left(d\tilde{a}^2 - \tilde{k}^2 A \right) - \frac{1}{g_1^2} F \wedge \star_4 F + \underbrace{\frac{1}{8\pi^2} \tilde{a}^2 \, \mathrm{Tr}(G \wedge G)}_{\mathrm{U}(1)\ \text{variant}} \right]$$

$$U(1): \; A \to A + d\eta', \; a^i \to a^i + k^i \eta'$$

$$\text{non-Abelian: } B \to B + D\eta$$

- Solution 1: **Triangle anomalies + Variance in instanton coupling = 0 (GS mech.)**

- Solution 2: **Triangle anomalies + Variance in instanton coupling + Variance in GCS terms = 0**

$$\boxed{\mathcal{S}_{sub}^{\mathrm{GCS}} = -\int \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{GCS}} A \wedge \Omega} \quad \boxed{\mathrm{d}\Omega = \mathrm{Tr}\left(G \wedge G\right)}$$

Axion Mixings (minimal)

Choose gauge to eat an axion

$$\mathcal{S}^{\text{full}}_{axion} = \int \Big[-\frac{f^2_{\tilde{a}^1}}{2} d\tilde{a}^1 \wedge \star_4 d\tilde{a}^1 - \frac{f^2_{\tilde{a}^2}}{2} \left(d\tilde{a}^2 - \tilde{k}^2 A \right) \wedge \star_4 \left(d\tilde{a}^2 - \tilde{k}^2 A \right) - \frac{1}{g_1^2} F \wedge \star_4 F$$
$$- \frac{1}{g_2^2} \mathrm{Tr}(G \wedge \star_4 G) + \frac{1}{8\pi^2} \left[\tilde{a}^1 + \tilde{a}^2 \right] \mathrm{Tr}(G \wedge G) - \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{GCS}} A \wedge \Omega + \ldots \Big]$$

$$\boxed{A \longrightarrow A + \frac{1}{\tilde{k}^2} d\tilde{a}^2}$$

$$\tilde{k}^2 = M_A \left(\frac{r_1}{\sqrt{\mathcal{G}_{11}}} \sin\varphi + \frac{r_2}{\sqrt{\mathcal{G}_{22}}} \cos\varphi \right) = r_1 k^1 + r_2 k^2$$

$$\mathcal{S}^{\text{full,unitary}}_{axion} = \int \Big[-\frac{f^2_{\tilde{a}^1}}{2} d\tilde{a}^1 \wedge \star_4 d\tilde{a}^1 - \frac{(f_{\tilde{a}^2} \tilde{k}^2)^2}{2} A \wedge \star_4 A - \frac{1}{g_1^2} F \wedge \star_4 F - \frac{1}{g_2^2} \mathrm{Tr}(G \wedge \star_4 G)$$
$$+ \frac{1}{8\pi^2} \tilde{a}^1 \mathrm{Tr}(G \wedge G) - \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{GCS}} A \wedge \Omega + A \wedge \star_4 \mathcal{J}_\psi$$
$$+ \frac{1}{8\pi^2} \frac{\left(\tilde{k}^2 + \mathcal{A}^{\mathrm{GCS}} + \mathcal{A}^{\mathrm{mix}} \right)}{\tilde{k}^2} \tilde{a}^2 \mathrm{Tr}(G \wedge G) + \ldots \Big] .$$

$$M_A^2 = \mathcal{G}_{11} \left(k^1 \right)^2 + \mathcal{G}_{22} \left(k^2 \right)^2$$

$$\mathcal{J}^\mu_\psi = \sum_i \left[(q^i_L) \overline{\psi}^i_L \gamma^\mu \psi^i_L + (q^i_R) \overline{\psi}^i_R \gamma^\mu \psi^i_R \right]$$

from chiral rotation of chiral fermions

Axion Mixings (minimal)

Integrate out the massive U(1)

EOM for massive U(1): $$-\frac{1}{g_1^2}d(\star_4 dA) - (f_{\tilde{a}_2}\tilde{k}^2)^2 \star_4 A = \frac{\mathcal{A}^{\rm GCS}}{8\pi^2}\Omega - \star_4 \mathcal{J}_\psi$$

2 possible sources

Take derivative

$$d(\star_4 \mathcal{J}_\psi) = -\frac{1}{8\pi^2}\mathcal{A}^{\rm mix}\, d\Omega = -\frac{1}{8\pi^2}\mathcal{A}^{\rm mix}\, {\rm Tr}(G\wedge G)$$

$$(f_{\tilde{a}^2}\tilde{k}^2)^2 d(\star_4 A) = \frac{\mathcal{A}^{\rm GCS} + \mathcal{A}^{\rm mix}}{\mathcal{A}^{\rm mix}} d(\star_4 \mathcal{J}_\psi)$$

Up to a total derivative

$$(f_{\tilde{a}^2}\tilde{k}^2)^2 d(\star_4 A) = \frac{\mathcal{A}^{\rm GCS} + \mathcal{A}^{\rm mix}}{\mathcal{A}^{\rm mix}} d(\star_4 \mathcal{J}_\psi)$$

Now

$$\mathcal{S}^{\rm full,unitary}_{axion} = \int \Big[-\frac{f^2_{\tilde{a}^1}}{2} d\tilde{a}^1 \wedge \star_4 d\tilde{a}^1 + \frac{1}{8\pi^2}\tilde{a}^1 {\rm Tr}(G\wedge G) + \frac{(\mathcal{A}^{\rm GCS}+\mathcal{A}^{\rm mix})^2}{2(\mathcal{A}^{\rm mix})^2}\frac{1}{(f_{\tilde{a}^2}\tilde{k}^2)^2}\mathcal{J}_\psi \wedge \star_4 \mathcal{J}_\psi + \ldots \Big]$$

4-point interaction suppressed by the U(1) mass

Axion Mixings (minimal)

Integrate out chiral fermions and non-Abelian gauge group

Similar to QCD

- Chiral fermions condensate into meson-like states.
- 4-point couplings give masses of fermions of $\sim \Xi^3 / f_{\tilde{a}^2}^2$
- Use non-linear sigma model techniques to integrate out heavy mesons.

condensate scale

Chiral Rotation

$$\mathcal{L}_{\rm fermion} = \overline{\psi}^i_L\, i\left(\partial\!\!\!/ - iq^i_L A\!\!\!/ - iB\!\!\!/^a T_a^{R_1^i}\right)\psi^i_L + \overline{\psi}^i_R\, i\left(\partial\!\!\!/ - iq^i_R A\!\!\!/ - iB\!\!\!/^a T_a^{R_2^i}\right)\psi^i_R$$

Under a chiral transformation of the type,

$$\psi^i_L \to e^{i\,q^i_L \tilde{a}^2}\psi^i_L, \qquad \psi^i_R \to e^{i\,q^i_R \tilde{a}^2}\psi^i_R,$$

$$\mathcal{L}^{\rm eff}_{\rm fermion} = \mathcal{L}_{\rm fermion} + \tilde{a}^2\left(\partial_\mu \mathcal{J}^\mu_\psi + \frac{1}{32\pi^2}\mathcal{A}^{\rm mix}\varepsilon^{\mu\nu\rho\sigma}{\rm Tr}(G_{\mu\nu}G_{\rho\sigma})\right)$$

Adler-Bell-Jackiw anomaly

Variance from the measure

Another Way to Generate the Sinusoidal Potential

- Way 2. **Gaugino condensates**: **break U(1) R** symmetry. **Supersymmetry obtains a NP correction**

Moduli dependent, assumed to be stabilized at some higher scale

$$\mathcal{W} = \mathcal{W}_{per} + A\, e^{-\frac{2\pi}{N}T}$$ superfield T as $t + i\,a$

rank of the non-Abelian group

axion

$$K(T,\overline{T}) = -3\ln(T+\overline{T})$$

F-term potential for N=1 SUGRA

$$V_{\text{axion}}(a) = \frac{8\pi}{N}\frac{\langle t\rangle}{\mathcal{T}^2}\,|A|\,|\mathcal{W}_{per}|\,e^{-2\frac{\pi}{N}t}\cos\left(\frac{2\pi}{N}a + i\gamma\right)$$

$\gamma = \mathrm{Arg}\left(\mathcal{W}_{per}A^*\right)$ $\quad\mathcal{T}$ the dimensionless volume of the internal space

Aligned Natural Inflation

N=2

$$\boxed{V(\Phi^1,\Phi^2) = \Lambda_1^4\left[1-\cos\left(\frac{n_1\Phi^1}{f_1}+\frac{n_2\Phi^2}{g_1}\right)\right] + \Lambda_2^4\left[1-\cos\left(\frac{m_1\Phi^1}{f_2}+\frac{m_2\Phi^2}{g_2}\right)\right], \qquad n_i,\, m_i \in \mathcal{Z}}$$

Perfect alignment $\dfrac{f_1/n_1}{g_1/n_2} = \dfrac{f_2/m_1}{g_2/m_2}$

Derivation from alignment $\alpha = g_2/m_2 - \dfrac{f_2/m_1}{f_1/n_1} g_1/n_2$

Along flat direction:

$f_{eff} \propto \dfrac{1}{\alpha}$

e.g. $f_1/n_1 = f_2/m_1 \ll g_1/n_2,\; g_2/m_2$

If want enhancement $\alpha \sim 10^{-2}\mathcal{O}(f_i,\, g_i)$, will require $\dfrac{n_2}{m_2} = \dfrac{99}{100}$

Too large integers!

Axion Bounds from String Compactifications

- Take weakly heterotic string for instance.

$$S_{bos} = M_s^8 \int d^{10}x\sqrt{-g}e^{-2\phi}(R - \frac{1}{2}|H_3|^2) - M_s^8 \int d^{10}x\sqrt{-g}e^{-2\phi}\frac{1}{2}|F_2|^2$$

$H_3 = dB_2$

Moduli from $B_{\mu\nu}$

dimensional reducing to 4d

$$M_s^8\, l_s^6 = M_P^2$$

$$\frac{f_a^2}{M_p^2} = 1$$

WCG and Swampland

- WGC (weak form) states that $\exists$ state with $\left(\frac{M}{Q}\right) \leq M_{Pl}$
  $\leadsto$ conjectured generalisation for 0-forms with $S_{inst} \leq \frac{M_{Pl}}{f}$
  Arkhani-Hamed-Motl-Nicolis-Vafa (2006)
- Consistent compactifications for

$$\begin{array}{ccc} \text{M-theory} & \overset{S,T}{\longleftrightarrow} & \text{Type IIB} \\ \mathcal{M}_{1,2} \times S^1_M \times \tilde{S}^1 \times X_6 & & \mathcal{M}_{1,2} \times S^1 \times X_6 \end{array}$$

  $\leadsto$ Constraints on effective axion decay constant by applying WGC on 5dim BH in M-theory
  Brown-Cottrell-Shiu-Soler (2015)
- Resolving ambiguity in defining effective axion decay constant
  $\leadsto$ possible axionic directions with $f > M_{Pl}$
  Bachlechner-Long-McAllister (2014/15), Junghans (2015)
- **BUT** in our simple model: 2 axions + 1 $U(1)$ + 1 instanton
  - apply WGC at appropriate scale $\Lambda > M_{\rm gauge}$
    $\Rightarrow \exists$ axionic direction with $f_2 = \frac{M_{\rm gauge}}{\tilde{k}^2} < M_{Pl}$ ✓WGC
  - Higher harmonics are still troublesome for $\perp$ axion

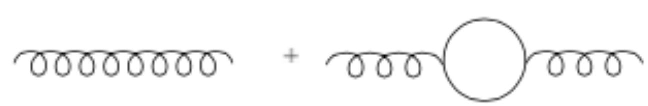
  Montero-Uranga-Valenzuela (2015)

Renormalization of the Planck Mass

- Running the Planck mass: $M_P \to M_P(\mu)$ for some scale μ

  $M_P(0) \sim 10^{19}\,\mathrm{GeV}$

- Graviton propagator:



- Integrating out (propagating) scalars, fermions and gauge bosons

$$M_P^2(\mu) \sim \frac{1}{G(\mu)} = \frac{1}{G(0)} - \frac{\mu}{12\pi}(n_0 + n_{1/2} - 4n_1)$$

n_i is the number of particles with spin-i

Moduli Stablization

- **A modulus field is a scalar field whose potential is vanishing** (thus massless).
- **The VEVs of those massless fields are moduli (sometimes we don't distinguish modulus and modulus field).**
- E.g. in models with 1 extra dimension compactified on a circle, the radius of this curled dimension is a modulus. The value of the radius sets the scale for the extra dimension, and affects the physics spectrum in lower dimension.
- E.g. the metric in the axion moduli space is field (modulus) dependent. For the toroidal case, the metric is diagonal with entries given by gauge coupling constants, determined by the volume of the internal cycles. The volumes are moduli.

Moduli Stablization

- ***Unstabilized moduli can cause serious problems!***
- **Predictions** of the theory **crucially depend on the VEVs of the moduli**. No potential means the VEVs can be arbitrary. Not much to predict.
- **Moduli can be time-dependent**, conflicting with observation.
- Massless scalars **can mediate long range force (~ gravity)**. Conflicting with Newton's law (**no such long range force exists**).
- **Polonyi problem**: in the early universe, the light scalars can have VEVs ~ Planck scale. Overclosure of the universe, etc.
- Different than a massless goldstone, **moduli exist with or without any symmetry**. Physics depends on the VEVs of moduli. Physically distinct vacua can be connected by varying moduli.

Moduli Stablization

- The **axion metric is moduli dependent**.
- In the work, ***assume the moduli (the volume, the shape of internal cycles, etc) have been stabilized at some higher energy scale***.
- How stabilized? Model dependent. Difficult.
- e.g. stabilized on orbifolds, Ruehle & Wieck, 1503.07183
- Rudelius 1409.5793, considered the moduli dependence of the metric, but assumed straight geodesics…

String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

Internal geometry $T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$ $\quad x^i \to x^i + R_1^{(i)},\ y^i \to y^i + R_2^{(i)}$

Symplectic basis of the homology (γ_i, δ^i)

Orientifold action $(x^i, y^i) \xrightarrow{\Omega\mathcal{R}} (x^i, -y^i)$

Bulk action -> metric on axion field space

$$\mathcal{K}_{ij} = \mathrm{diag}\left(u_1 u_2 u_3, \frac{u_1}{u_2 u_3}, \frac{u_2}{u_1 u_3}, \frac{u_3}{u_1 u_2}\right) \qquad u_i = R_2^{(i)}/R_1^{(i)}$$

three-cycle Π_x wrapped by a D6$_x$-brane $\quad \Pi_x = r_x^i\, \gamma_i + s_x^i\, \delta^i$

String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

Cohomology basis

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3, & \beta^0 &= dy^1 \wedge dy^2 \wedge dy^3, \\ \alpha_1 &= dx^1 \wedge dy^2 \wedge dy^3, & \beta^1 &= dy^1 \wedge dx^2 \wedge dx^3, \\ \alpha_2 &= dy^1 \wedge dx^2 \wedge dy^3, & \beta^2 &= dx^1 \wedge dy^2 \wedge dx^3, \\ \alpha_3 &= dy^1 \wedge dy^2 \wedge dx^3, & \beta^3 &= dx^1 \wedge dx^2 \wedge dy^3, \end{aligned}$$

Tadpole

$$\sum_x N_x(\Pi_x + \Pi'_x) = 4\Pi_{O6}.$$

String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

Setup: a stack of D6_a branes and a single D6_b brane

$$U(N_a) : \Pi_a = r_a^2 \gamma_2 + r_a^3 \gamma_3 + s_a^2 \delta^2 + s_a^3 \delta^3, \qquad U(1)_b : \Pi_b = r_b^i \gamma_i .$$

$$\begin{aligned} S_{axion} = \int \Big[& -\frac{1}{2\ell_s^2} \sum_{i=0,1} \mathcal{K}_{ii} da^i \wedge \star_4 da^i - \frac{1}{2\ell_s^2} \sum_{l=2,3} \mathcal{K}_{ll} (da^l - N_a s_a^l A_a) \wedge \star_4 (da^l - N_a s_a^l A_a) \\ & + \frac{1}{8\pi^2} \left(r_a^2 a^2 + r_a^3 a^3 \right) \mathrm{Tr}(G_a \wedge G_a) + + \frac{1}{8\pi^2} \left(r_a^2 a^2 + r_a^3 a^3 \right) N_a F_a \wedge F_a \\ & + \frac{1}{8\pi^2} \left(\sum_{i=0}^{3} r_b^i a^i \right) (F_b \wedge F_b) \Big] . \end{aligned}$$

Put O6 along the even cycle γ_0

two decoupled axion systems (a^0, a^1) and (a^2, a^3)

Focus

String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

- In the system with axions a_2 and a_3, the uneaten axion has an effective decay constant

$$f_{\tilde{a}_1} = \sqrt{\frac{u_2 u_3}{u_1}} \frac{\sqrt{(u_2)^2 (s_a^2)^2 + (u_3)^2 (s_a^3)^2}}{|r_a^2 s_a^3 (u_3)^2 - r_a^3 s_a^2 (u_2)^2|} M_s$$

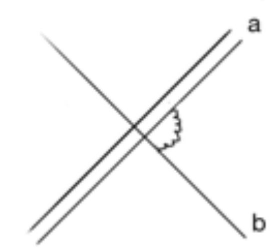
- Can be large if $r_a^2 s_a^3 (u_3)^2 \simeq r_a^3 s_a^2 (u_2)^2$ $\qquad u_i = R_2^{(i)} / R_1^{(i)}$

- **Tadpole condition** used to determine integer coefficients

String Theory Embedding

Explicit D6-brane model to realize super-Planckian excursion

2 stacks of D6's wrapping ***non-factorisable*** 3-cycles



Overview of Chiral Spectrum for Non-factorizable D6-branes

sector	$SU(N_b)_{(Q_a,Q_b)}$	multiplicity
ab	$(\mathbf{N_a})_{(1,-1)}$	$\lvert -s_a^2 r_b^2 - s_a^3 r_b^3 \rvert$
ab'	$(\mathbf{N_a})_{(1,1)}$	$\lvert -s_a^2 r_b^2 - s_a^3 r_b^3 \rvert$
aa'	$(\mathbf{Anti}_a)_{(2,0)}$	$\lvert -r_a^2 s_a^2 - r_a^3 s_a^3 \rvert$
aa'	$(\mathbf{Sym}_a)_{(2,0)}$	$\lvert -r_a^2 s_a^2 - r_a^3 s_a^3 \rvert$

Internal space

$T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$

One solution

$N_a = 3.$ $\quad r_a^2 = r_a^3 = 1, \quad s_a^2 = 2,\ s_a^3 = 3, \quad r_b^0 = 16,\ r_b^1 = 0, \quad r_b^2 = r_b^3 = -3.$

chiral spectrum $\quad 15 \times (\mathbf{3})_{(1,-1)} + 15 \times (\mathbf{3})_{(1,1)} + 5 \times (\mathbf{3}_A)_{(-2,0)} + 5 \times (\overline{\mathbf{6}}_S)_{(-2,0)}$

u_3/u_2 asymptotes to the value $\sqrt{2}/\sqrt{3}$ $\quad M_s \sim 10^{17}$ GeV $\quad f_{\tilde{a}^1} \approx \frac{M_s}{3} \times 10^3 \sim 10 M_{Pl}.$