Observables in Gauge/Gravity Duality

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Target:

Use the gauge/gravity duality.

$\Downarrow$

Study the heavy quark observables in different theories.

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Look for universal properties among them.

Two such examples are reviewed here.

Outline

Introduction

- To examine certain properties of any theory we need to interact with it and compute expectation values. We do the same in gauge/gravity correspondence.
- One step further is to extract information for a large class of theories using common properties of the observables!
- In deconfined phase the dual theories have common characteristics, strongly coupled; no susy; no confinement. And differences: E.g. Number flavors, number of degrees of freedom etc. These may not, or weakly affect the observables in certain comparison schemes.

Gauge/Gravity Duality: Several Examples

- The initial AdS/CFT correspondence is the harmonic oscillator of the gauge/gravity dualities.
- Since its discovery, there is an extensive research aiming to construct more realistic gauge/gravity dualities.

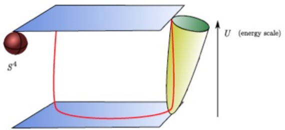
✓ Less Supersymmetric Dualities. E.g. β deformation: $AdS_5 \times \tilde{S}^5$, $\mathcal{N} = 1$ susy.

✓ Additional branes to the original theory.
E.g. Inclusion of fundamental degrees of freedom with D3/D7 systems.
E.g. Inclusion of Anisotropy with anisotropically distributed heavy branes.

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

Very interesting theories, several new features!

✓ Broken conformal symmetry, confinement.
Example: $D4$ Witten model.

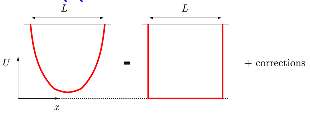


(Fig: 0708.1502)

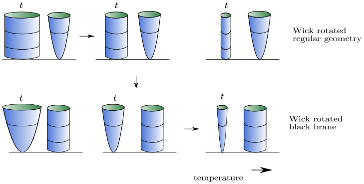
$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu} + f(u)dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \quad f(u) = 1 - \left(\frac{u_k}{u}\right)^3$$

The potential of the static heavy meson is linear:

$$V_{Q\bar{Q}} = \sigma L + \mathcal{O}(e^{-L}) \quad .$$



✓ Finite temperature. Presence of Black hole.



Example: An anisotropic black hole: *(Mateos, Trancanelli, 2011)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\, d\Omega_{S^5}^2 \,.$$

The anisotropic parameter is α appears in the axion $(\chi = \alpha x_3)$. In high temperatures, $T \gg \alpha$:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 F_1(u, u_h) \,,$$

$$\mathcal{B}(u) = 1 - \alpha^2 B_1(u, u_h), \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

General Theory for Observables

- Assume the existence of the gravity dual of a theory.
- For most of the observables we can work in full generality with:

$$ds^2 = g_{00}(u)dx_0^2 + \sum g_{ii}(u)dx_i^2 + g_{uu}(u)du^2 + \text{internal space}$$

Where: x_0 is time, x_i forms the space, and u is the holographic radial direction.

- The background may have RR fluxes as well a non-trivial dilaton.

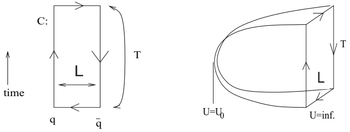
Static Potential

Warm-up observable:

- The string world-sheet (τ, σ) of the following form.

String Configuration

$$x_0 = \tau, \quad x_1 = \sigma, \quad u = u(\sigma)\ , \quad u(0) = u(L) = u_{Boundary}$$



The solution to Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-g_{00}(g_{11} + g_{uu} u'^2)}$$

is a catenary shape w-s with u_0 being the turning point.

In general the length of the two endpoints of the string on the boundary is given by

$$L = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu} c_0^2}{(g_{00} g_{11} + c_0^2) g_{11}}}$$

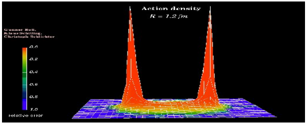
$c_0 \sim \sqrt{g_{00} g_{11}}_{(u_0)}$. L should be inverted as $u_0(L)$ to find the normalized energy of the string is

$$2\pi\alpha' V(L) = c_0 L + 2\left[\int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}} \left(\sqrt{1 + \frac{c_0^2}{g_{11} g_{00}}} - 1\right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}}\right].$$

(Sonnenschein 2000, ...)

Width of the Chromoelectric flux tube

The chromoelectric field energy density between the $Q\bar{Q}$ is confined.



(*Fig:Bali, Schilling, Schlichter, 1995*)

To measure it we use a small probe Wilson loop $P(c)$, transverse to the WL at distance Δx_3 that corresponds to the heavy quark pair $W(C)$

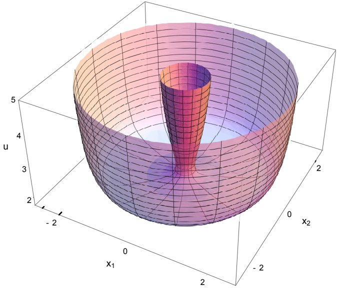
$$\mathcal{S}(x) = \frac{\langle W(C)P(c)\rangle - \langle W(C)\rangle\langle P(c)\rangle}{\langle W(C)\rangle} \, .$$

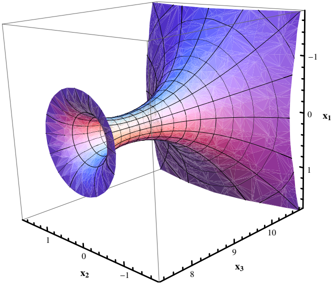
(*Lüsher, Munster, Weisz 1980*)

The mean square width of the flux tube is then defined as

$$w^2 = \frac{\int d(\Delta x_3)\Delta x_3^2 \; \mathcal{S}}{\int d(\Delta x_3) \; \mathcal{S}} \, .$$

Holographically we compute the connected minimal surface between two circles with radii $R \gg r_0$.





The system of equations in static gauge: $\theta = \tau,\ x_3 = \sigma; (x_1, x_2) \to (r, \theta)$

$$r'' - hr = 0\ ,$$

$$2u'' + u'^2 \partial_u (\ln f) - r^2 \frac{\partial_u h}{f} = 0\ ,$$

$$r'^2 + f u'^2 = hr^2 - 1\,.$$

with, $r(\sigma)$ the radii of the circles, $u(\sigma)$ the holographic coordinate and

$$h(u) := \frac{g_{11}^2}{c^2}, \qquad f(u) := \frac{g_{uu}}{g_{11}}\ .$$

Using the properties of the confining gravity dual metric for any confining background:

$$S = \frac{g_{11}(u_k)}{2}\left(\frac{\Delta x_3}{\sqrt{h}} + R^2 - r_0^2 + \frac{1}{2}\left(1 - \epsilon^{-2\sqrt{h}\Delta x_3}\right)\right) \simeq \sigma\left(\frac{\Delta x_3^2}{\log\frac{R}{r_0}} + R^2 - r_0^2\right),$$

Resulting the logarithmic broadening with the $Q\bar{Q}$ length R:

$$w^2 \simeq \frac{1}{2\pi\sigma}\log\frac{R}{r_0} \ ,$$

Universal feature for any confining holographic theory! *(D.G.,Irges 2015)*

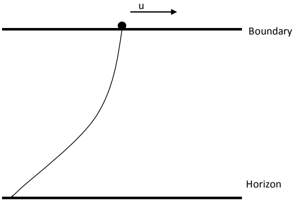
✓ Logarithmic Broadening in Lattice Computations. e.g. *(Gliozzi, Pepe, Wiese, 2010;...)*

Deconfined phase

The dynamics and the interactions of the heavy quark can be described by a diffusion treatment. The thermal momentum of the quark is $p_{th}^2 \simeq 3 m_Q T \gg T^2$. The momentum transfer of the medium is $Q^2 \simeq T^2$. So Brownian motion of the heavy quark in a light particle fluid:

$$\frac{dp}{dt} = F_{drag} + F(t) \; .$$

The drag force F_{drag} of a single quark moving in the plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. (*Gubser, 2006*)



The drag force of a quark moving along the x_1 direction, for any background is given by the momentum flowing from the boundary to the bulk

$$F_{drag} = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{11}}}{(2\pi)}\Big|_{u=u_0}$$

where u_0 is given by

$$\left(g_{uu}(g_{00} + g_{11}v^2)\right)\,|_{u=u_0} = 0\ .$$

u_0 is the of the induced worldsheet metric.
Quick Check: For $v = 0 \Rightarrow$ straight world sheet, no horizon.

- The 'effective world-sheet temperature' is

$$T_{ws}^2 = \left|\frac{1}{16\pi^2}\frac{1}{g_{00}g_{uu}}(g_{00}\,g_{11})'\left(\frac{g_{00}}{g_{11}}\right)'\right|\Big|_{u=u_0}\ .$$

In near horizon Dp black brane geometries $T_{ws} < T$(heat bath temperature). (*Nakamura, Ooguri 2013*)

Momentum Broadening

The $F(t)$ is the factor that causes the momentum broadening, which leads to

$$\frac{\left\langle p_{L,T}^2 \right\rangle}{\mathcal{T}} = 2\kappa_{L,T}$$

$\kappa =$ Mean Squared Momentum Transfer per Time.

- The index L refers to the direction along the motion of quark, the index T is the direction transverse to the velocity of quark.
- In strong coupling limit for a quark moving along x_1 direction, these fluctuations are introduced to the Wilson line

$$t = \tau, \quad u = \sigma, \quad x_1 = v\ t + \xi(\sigma) + \delta x_1(\tau, \sigma), \quad x_{2,3} = \delta x_{2,3}(\tau, \sigma)\ .$$

$\delta x_1(\tau, \sigma)$: Longitudinal fluctuation,
$\delta x_{2,3}(\tau, \sigma)$: Transverse fluctuations .
We compute the effect on the Nambu-Goto action and specify the coefficients.

Their ratio can be simplified to

$$\frac{\kappa_L}{\kappa_T} = \frac{1}{g_{11} g_{22}} \frac{(g_{00} g_{11})'}{(g_{00}/g_{11})'}\bigg|_{u=u_0}$$

Reminer: Quark moves along the direction x_1, and the transverse direction to motion for the momentum broadening is x_2.

•For any isotropic theory $g_{11} = g_{22}$ and $g_{00} = g_{00,bh}\ g_{22}\ f$, we prove $\kappa_L > \kappa_T$.

• This is a Universal Inequality independent of the background used!

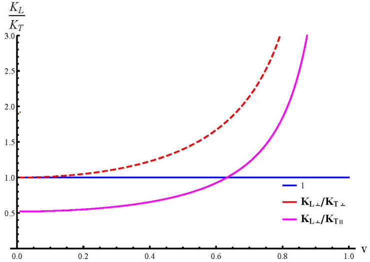
(D.G, Soltanpanahi, 2013a; Gursoy, Kiritsis, Mazzanti, Nitti, 2010)

• The only possibility to have violation of the inequality is in the anisotropic theories!

• In fact the motion of the quark in the axion deformed anisotropic theory violates the inequality! *(D.G, Soltanpanahi, 2013b)*

Examining the quark motion in the space dependent axion anisotropic background.

• $\kappa_L < \kappa_T$ when the motion of the quark is along the transverse to the anisotropic direction.



Conclusions

Working with a large class of theories we obtain universal behaviors.

- Logarithmic flux tube broadening, in confining theories.
- The Universal Langevin coefficients inequality $\kappa_L > \kappa_T$ proved to hold for isotropic backgrounds is violated for the anisotropic theories!

  Similar treatment:

- Non-Integrability for large class of backgrounds. *(D.G, Sfetsos 2014)*.
- k-strings = fundamental strings with effective string tension for large class of theories. *(D.G, 2015)*.

Thank you