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Matter-Moduli couplings in SYM theories with magnetized extra dimensions

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With H. Abe, T. Horie, T. Kobayashi and H. Ohki

Moduli (Axion) impacts on phenomenologies

- Yukawa couplings as 4D effective couplings
- SUSY spectra
- Moduli (Axion) Inflation
- Decay into matter fields

⋮

Matter-Moduli couplings

We need systems to predict explicit forms of the moduli couplings

➡ **Magnetized toroidal compactifications of SYM theories**

Magnetic fluxes on tori

- Generations of chiral fermions (C. Bachas '95)

➡ MSSM-like models derived from SYM theories
(4+2n)-dimensional U(N) SYM theories

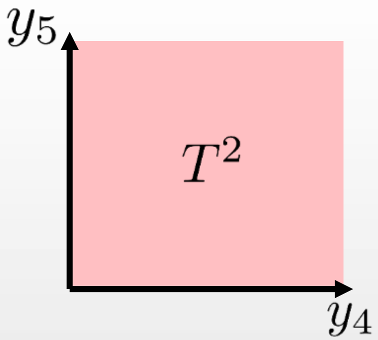
ex.) $$S = \int d^{10}x\sqrt{-G}\left\{-\frac{1}{4g^2}\mathrm{tr}\left(F^{MN}F_{MN}\right)+\frac{i}{2g^2}\mathrm{tr}\left(\bar{\lambda}\Gamma^M D_M\lambda\right)\right\}$$

- Easy to calculate 4D effective action analytically,

identifying its dependence on moduli fields

6D U(N) SYM theory (review of magnetized SYM)

- Compactified on $M^4 \times T^2$ with magnetic fluxes



Constant (Abelian) magnetic flux

$$F_{45} = 2\pi M \quad (M \in \boldsymbol{Z})$$

Gauge potential

$$A_4 = 0 \quad A_5 = M y_4$$

6D vector → 4D vector + 4D scalars

6D spinor → SO(4) × SO(2) : $\psi_{4D} \times \psi_{\pm}$

- KK expansion

$$A_M(x^\mu, y^m) = \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y^m)$$

$$\lambda(x^\mu, y^m) = \sum_n \chi_n(x^\mu) \times \psi_n(y^m)$$

Gauge symmetry breaking on the magnetized torus

The abelian magnetic flux of the form

$$F_{45} = 2\pi M = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b} \end{pmatrix}$$

breaks $U(N)$ down to $U(N_a) \times U(N_b)$

$$\psi_{\pm} = \begin{pmatrix} \psi_{\pm}^{aa} & \psi_{\pm}^{ab} \\ \psi_{\pm}^{ba} & \psi_{\pm}^{bb} \end{pmatrix}$$

Bi-fundamental matters

$(N_a, \bar{N}_b)$ $(\bar{N}_a, N_b)$

Zero-modes on a magnetized torus

- Dirac equation

$$i\Gamma_m D^m \psi_0(y) = 0$$

$$\downarrow$$

$$\left[\partial_4 + i\partial_5 + 2\pi\left(M_a - M_b\right) y_4\right]\psi_+^{ab} = 0$$

$$\left[\partial 4 - i\partial_5 - 2\pi\left(M_a - M_b\right) y_4\right]\psi_-^{ab} = 0$$

If $M_{ab} \equiv M_a - M_b > 0$

- ψ_+^{ab} has degenerate zero-modes, while ψ_-^{ab} does none
- $|M_{ab}|$ independent zero-mode solutions

→ Generations of chiral fermions

C. Bachas '95 ; D. Cremades, L. E. Ibanez & F. Marchesano '04

Zero-mode wavefunctions

(D. Cremades, L. E. Ibanez & F. Marchesano '04)

$$\Theta^{j,M_{ab}}(y_4, y_5) = \mathcal{N}_j e^{-M_{ab}\pi y_4^2} \cdot \vartheta \begin{bmatrix} j/M_{ab} \\ 0 \end{bmatrix} \left(-M_{ab} i \left(y_4 + i y_5\right), M_{ab} i\right)$$

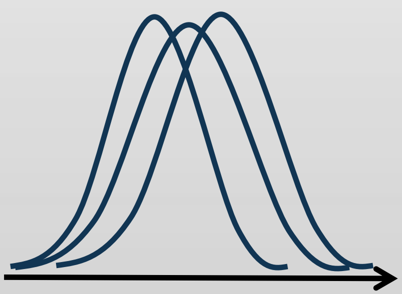
Jacobi-theta function

$$j = 1, 2, \ldots, |M_{ab}|$$

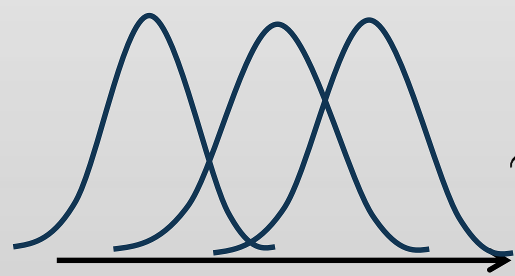
$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{l \in \mathbb{Z}} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}.$$

- The action integral can be performed analytically
- Quasi-localizations (N. Arkani-Hamed & M. Schmaltz '00)

$$y_{IJK} = \int dy^2 \; \Theta^{I,M_{ab}}(y) \Theta^{J,M_{bc}}(y) \Theta^{I,M_{ca}}(y)$$



$\sim \mathcal{O}(1)$



$\sim \mathcal{O}(10^{-5})$

10D SYM theories
in superspace formulation

Superfield description of 10D SYM theories

$$S = \int dx^{10}\sqrt{-G}\left[-\frac{1}{4g^2}\mathrm{tr}\left(F^{MN}F_{MN}\right) + \frac{i}{2g^2}\mathrm{tr}\left(\bar{\lambda}\Gamma^M D_M\lambda\right)\right]$$

- 4D N=1 decomposition $i\ :\ 1,2,3$

10D vector field $A_M = (A_\mu,\ A_i)$

$$A_i \equiv -\frac{1}{\mathrm{Im}\tau_i}\left(\bar{\tau}_i A_{2+2i} - A_{3+2i}\right)$$

→ 4D vector+ (complex scalar) × 3

10D Majorana-Weyl spinor:

$$\lambda = (\lambda_0,\ \lambda_i) \qquad \lambda_0 = \lambda_{+++} \quad \lambda_1 = \lambda_{+--} \quad \lambda_2 = \lambda_{-+-} \quad \lambda_3 = \lambda_{--+}$$

→ 4D Wely spinor × 4

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4D N=1 vector multiplet $V = \{A_\mu,\ \lambda_0\}$

4D N=1 chiral multiplets $\phi_i = \{A_i,\ \lambda_i\}$ $i\ :\ 1,2,3$

N. Marcus, A. Sagnotti and W. Siegel ’83
N. Arkani-Hamed, T. Gregoire and J. Wacker ’02

Superfield description of 10D SYM theories

$$V \equiv -\theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}\bar{\theta}\theta\lambda_0 - i\theta\theta\bar{\theta}\bar{\lambda}_0 + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D$$

$$\phi_i \equiv \frac{1}{\sqrt{2}}A_i + \sqrt{2}\theta\lambda_i + \theta\theta F_i$$

$$S = \int dx^{10}\sqrt{-G}\left[-\frac{1}{4g^2}\mathrm{tr}\left(F^{MN}F_{MN}\right) + \frac{i}{2g^2}\mathrm{tr}\left(\bar{\lambda}\Gamma^M D_M\lambda\right)\right]$$

$$= \int d^{10}X\sqrt{-G}\left[\int d^4\theta\,\mathcal{K} + \left\{\int d^2\theta\left(\frac{1}{4g^2}\mathcal{W}^\alpha\mathcal{W}_\alpha + \mathcal{W}\right) + \text{h.c.}\right\}\right]$$

$$\mathcal{K} = \frac{2}{g^2}h^{\bar{i}j}\mathrm{Tr}\left[\left(\sqrt{2}\bar{\partial}_{\bar{i}} + \bar{\phi}_{\bar{i}}\right)e^{-V}\left(-\sqrt{2}\partial_j + \phi_j\right)e^V + \bar{\partial}_{\bar{i}}e^{-V}\partial_j e^V\right] + \mathcal{K}_{\mathrm{WZW}}$$

$$\mathcal{W} = \frac{1}{g^2}\epsilon^{\mathrm{ijk}}e_{\mathrm{i}}{}^i e_{\mathrm{j}}{}^j e_{\mathrm{k}}{}^k\mathrm{Tr}\left[\sqrt{2}\,\phi_i\left(\partial_j\phi_k - \frac{1}{3\sqrt{2}}[\phi_j,\phi_k]\right)\right]$$

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$$

N. Marcus, A. Sagnotti and W. Siegel '83
N. Arkani-Hamed, T. Gregoire and J. Wacker '02

4D effective action in superfield description

- 4D N=1 SUSY configuration of magnetic fluxes : $\langle D \rangle = \langle F_i \rangle = 0$
- Zero-mode equations for the superfields

$$\left[\bar{\partial}_i + \frac{\pi}{2\mathrm{Im}\tau^{(i)}} \left(M_a^{(i)} - M_b^{(i)} \right) z_i \right] \phi_j^{(i)\,ab} = 0 \qquad \text{for } i = j$$

$$\left[\partial_i - \frac{\pi}{2\mathrm{Im}\tau^{(i)}} \left(M_a^{(i)} - M_b^{(i)} \right) \bar{z}_i \right] \phi_j^{(i)\,ab} = 0 \qquad \text{for } i \neq j$$

- Dimensional reduction in the same manner
- 4D effective action described in the superspace formulation

Moduli dependence of 4D effective action

The 4D effective action contains parameters $\{g, R_i, \tau_i\}$ $(i = 1,2,3)$

Gauge coupling constant	g	⬌	S : Dilaton
Torus sizes	R_i		T_i : Kähler moduli
Torus shapes	τ_i		U_i : Complex structure moduli

$$\mathrm{Re}\langle S\rangle = e^{-\langle\phi\rangle}\frac{\mathcal{A}_1}{\alpha'}\frac{\mathcal{A}_2}{\alpha'}\frac{\mathcal{A}_3}{\alpha'}, \qquad \mathrm{Re}\langle T_i\rangle = e^{-\langle\phi\rangle}\frac{\mathcal{A}_i}{\alpha'}, \qquad \langle U_i\rangle = i\bar{\tau}_i$$

where

$$\mathcal{A}_i = (2\pi R_i)^2 \mathrm{Im}\,\tau_i, \qquad g^2 = e^{\langle\phi\rangle}\alpha'^3$$

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Identify the moduli dependence of the 4D effective action

Kähler metrics and Yukawa couplings

- Functions of the moduli superfields

$$Z^{\bar{i}j}_{\mathcal{I}_{ab}}(S,T,U) = \delta^{\bar{i}j}\left(\frac{T_j+\bar{T}_{\bar{j}}}{2}\right)^{-1}\left(\prod_r \frac{U_r+\bar{U}_{\bar{r}}}{2}\right)^{-1/2}$$
$$\times\frac{1}{2^{5/2}}\left(\frac{|M^{(j)}_{ab}|}{\prod_{r\neq j}|M^{(r)}_{ab}|}\right)^{1/2}\exp\left[-\sum_r \frac{4\pi}{U_r+\bar{U}_{\bar{r}}}\frac{(\mathrm{Im}\,\zeta^{(r)}_{ab})^2}{M^{(r)}_{ab}}\right]$$

$$\lambda^{ijk}_{\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}(U) = -\frac{1}{3}\epsilon^{\mathrm{ijk}}\delta_{\mathrm{i}}{}^{i}\delta_{\mathrm{j}}{}^{j}\delta_{\mathrm{k}}{}^{k}\lambda^{(1)}_{ab,c}\lambda^{(2)}_{bc,a}\lambda^{(3)}_{ca,b}$$

$$\lambda^{(r)}_{ab,c} = \sum_{m=1}^{M^{(r)}_{ab}}\delta_{I^{(r)}_{bc}+I^{(r)}_{ca}-mM^{(r)}_{bc},\,I^{(r)}_{ab}}$$
$$\times\vartheta\begin{bmatrix}\frac{M^{(r)}_{bc}I^{(r)}_{ca}-M^{(r)}_{ca}I^{(r)}_{bc}+mM^{(r)}_{bc}M^{(r)}_{ca}}{M^{(r)}_{ab}M^{(r)}_{bc}M^{(r)}_{ca}}\\ 0\end{bmatrix}\left(\bar{\zeta}^{(r)}_{ca}M^{(r)}_{bc}-\bar{\zeta}^{(r)}_{bc}M^{(r)}_{ca},\,iU_rM^{(r)}_{ab}M^{(r)}_{bc}M^{(r)}_{ca}\right)$$

- Universal gauge kinetic functions : $f_a = S$

(4+2n)-dimensional SYM theories and their mixtures

Example 1

- 6D U(M) SYM+10D U(N) SYM (motivated by D5-D9 brane systems)
- 4D effective action can be derived in the same manner,

  identifying its dependence on the moduli fields

$$\mathrm{Re}\langle S\rangle = e^{-\langle\phi\rangle}\frac{\mathcal{A}_1}{\alpha'}\frac{\mathcal{A}_2}{\alpha'}\frac{\mathcal{A}_3}{\alpha'}, \quad \mathrm{Re}\langle T_i\rangle = e^{-\langle\phi\rangle}\frac{\mathcal{A}_i}{\alpha'}, \quad \langle U_i\rangle = i\bar{\tau}_i$$

$$g_{6\mathrm{D}}^2 = e^{\langle\phi\rangle}\alpha', \qquad g_{10\mathrm{D}}^2 = e^{\langle\phi\rangle}{\alpha'}^3$$

- New forms of moduli couplings;

  6D U(M) adjoint and bi-fundamentals $(M, \bar{N})$
- Non-universal gauge kinetic functions : $f_a = p_a S + q_a T_i$

Example 2

- 4D U(M) SYM+8D U(N) SYM (motivated by D3-D7 brane systems)
- T-dual to the previous SYM systems

$$\mathrm{Re}\langle S\rangle = e^{-\langle\phi\rangle}, \quad \mathrm{Re}\langle T_i\rangle \underset{(i\neq j\neq k\neq i)}{=} e^{-\langle\phi\rangle}\frac{\mathcal{A}_j}{\alpha'}\frac{\mathcal{A}_k}{\alpha'}, \quad \langle U_i\rangle = i\bar{\tau}_i$$

- Two sectors can be sequestered spatially

Realistic model

Realistic magnetic configurations in 10D SYM theories

- Preserved 4D N=1 SUSY
- The SM gauge group : $U(8) \to SU(3)_C \times SU(2)_L \times U(1)_Y$
- The MSSM field contents (3 gen. of the quarks/leptons, Higgs)

$$
\begin{aligned}
Z_{I\bar{I}}^{(Q_L)} &= \delta_{I\bar{I}} \frac{1}{\sqrt{3}} \left(T_2+\bar{T}_2\right)^{-1} \left(U_1+\bar{U}_1\right)^{-1/2} \left(U_2+\bar{U}_2\right)^{-1/2} \exp \frac{4\pi \left(\mathrm{Im}\,\zeta_{Q_L}\right)^2}{3\left(U_1+\bar{U}_1\right)}, \\
Z_{J\bar{J}}^{(Q_R)} &= \delta_{J\bar{J}} \frac{1}{\sqrt{3}} \left(T_3+\bar{T}_3\right)^{-1} \left(U_1+\bar{U}_1\right)^{-1/2} \left(U_3+\bar{U}_3\right)^{-1/2} \exp \frac{4\pi \left(\mathrm{Im}\,\zeta_{Q_R}\right)^2}{3\left(U_1+\bar{U}_1\right)}, \\
Z_{K\bar{K}}^{(H)} &= \delta_{K\bar{K}} \sqrt{6} \left(T_1+\bar{T}_1\right)^{-1} \left\{ \prod_{i=1}^{3} \left(U_i+\bar{U}_i\right)^{-1/2} \right\} \exp \frac{-4\pi \left(\mathrm{Im}\,\zeta_{H}\right)^2}{6\left(U_1+\bar{U}_1\right)},
\end{aligned}
$$

$$
\lambda_{IJK}^{(Q_R)} = \sum_{m=1}^{6} \delta_{I+J+3(m-1),K}\, \vartheta \begin{bmatrix} \frac{3(I-J)+9(m-1)}{54} \\ 0 \end{bmatrix} \left(3\left(\bar{\zeta}_{Q_L} - \bar{\zeta}_{Q_R}\right), 54iU_1\right)
$$

- (almost) Unique ansatz up to global factors
- The consistent MSSM spectrum

(Abe, Kobayashi, Ohki, Oikawa & KS ’13; Abe, Kawamura & KS ’14)

Summary

- Magnetized SYM system ;

  SM gauge group, gen. of chiral fermion, hierarchical Yukawas, …
- Analytical derivation of the 4D effective action
- Manifest forms of matter-moduli couplings in MSSM-like models

Future prospects

- SUSY spectrum in the light of LHC run 2
- Decay (& production) process in the early universe
- Moduli stabilization, …
- Other manifold (magnetized sphere, …)