Nonperturbative Beta Function of the SU(3) Gauge Theory with Nf=10 Massless Fermions

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Outline

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- **Finite Volume Gradient Flow Scheme**
- **Simulation of SU(3) gauge theory with Nf=10**
- **Renormalized Couplings**
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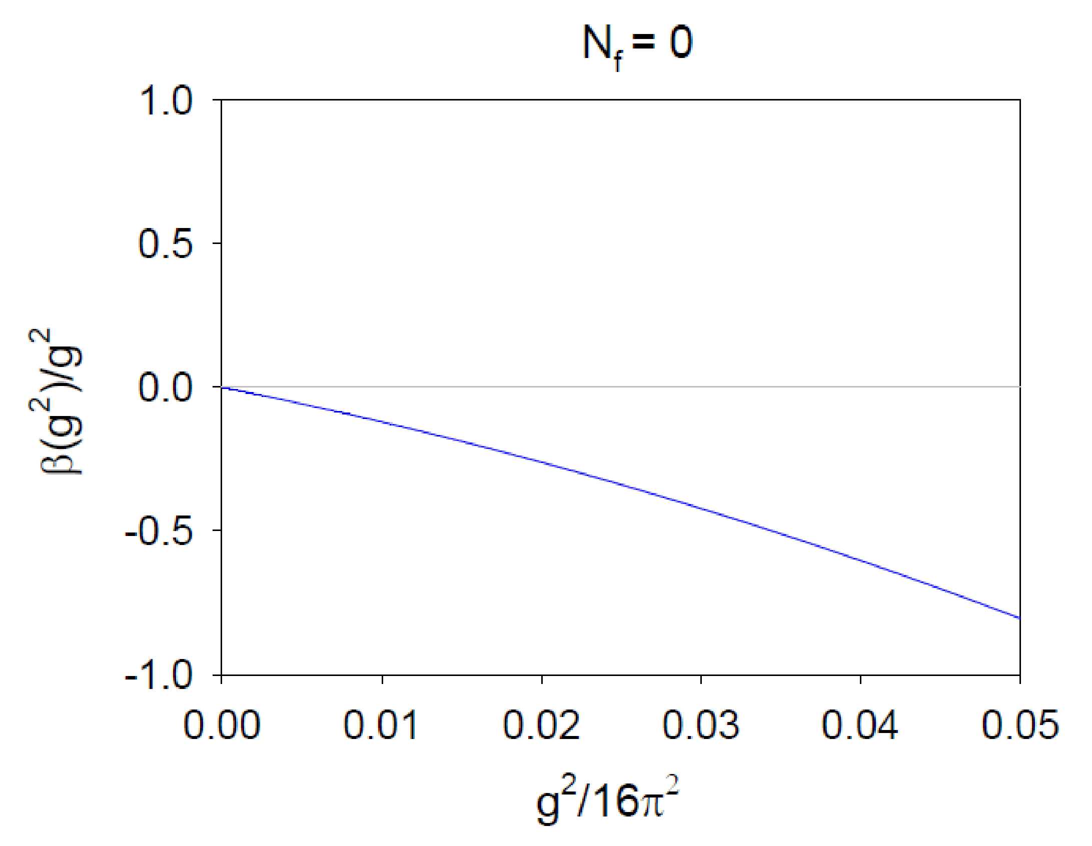

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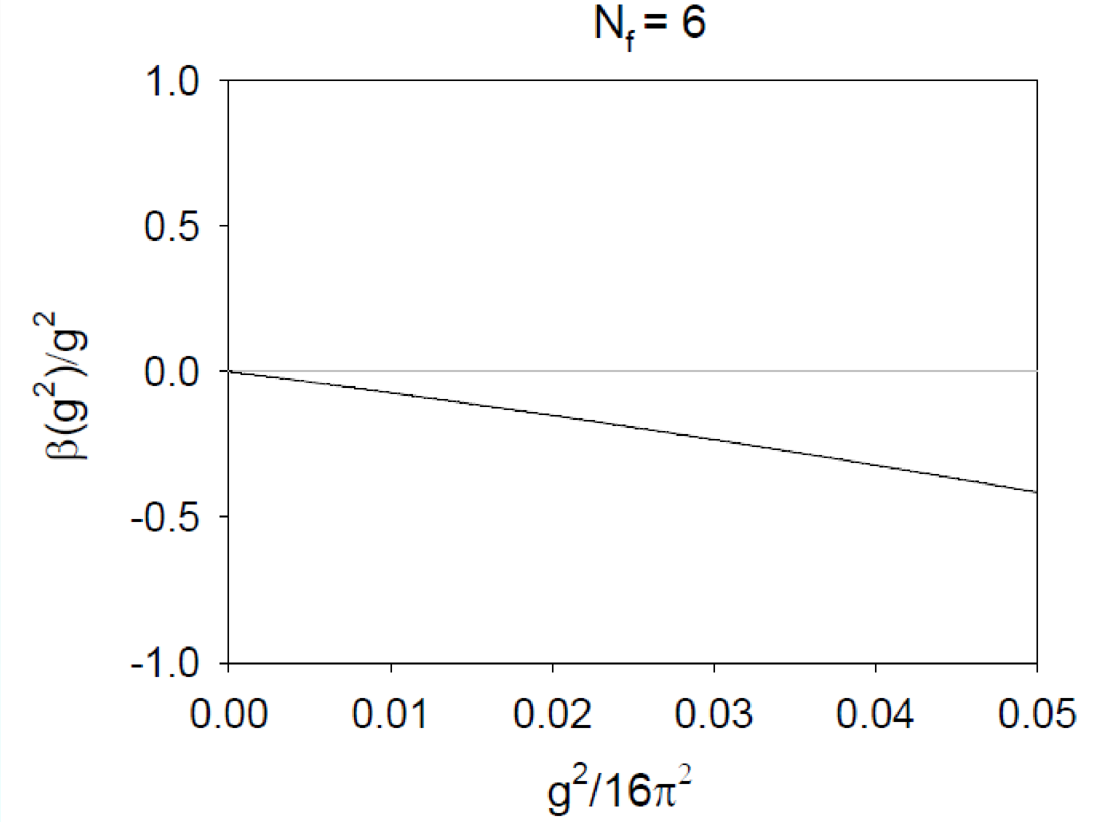

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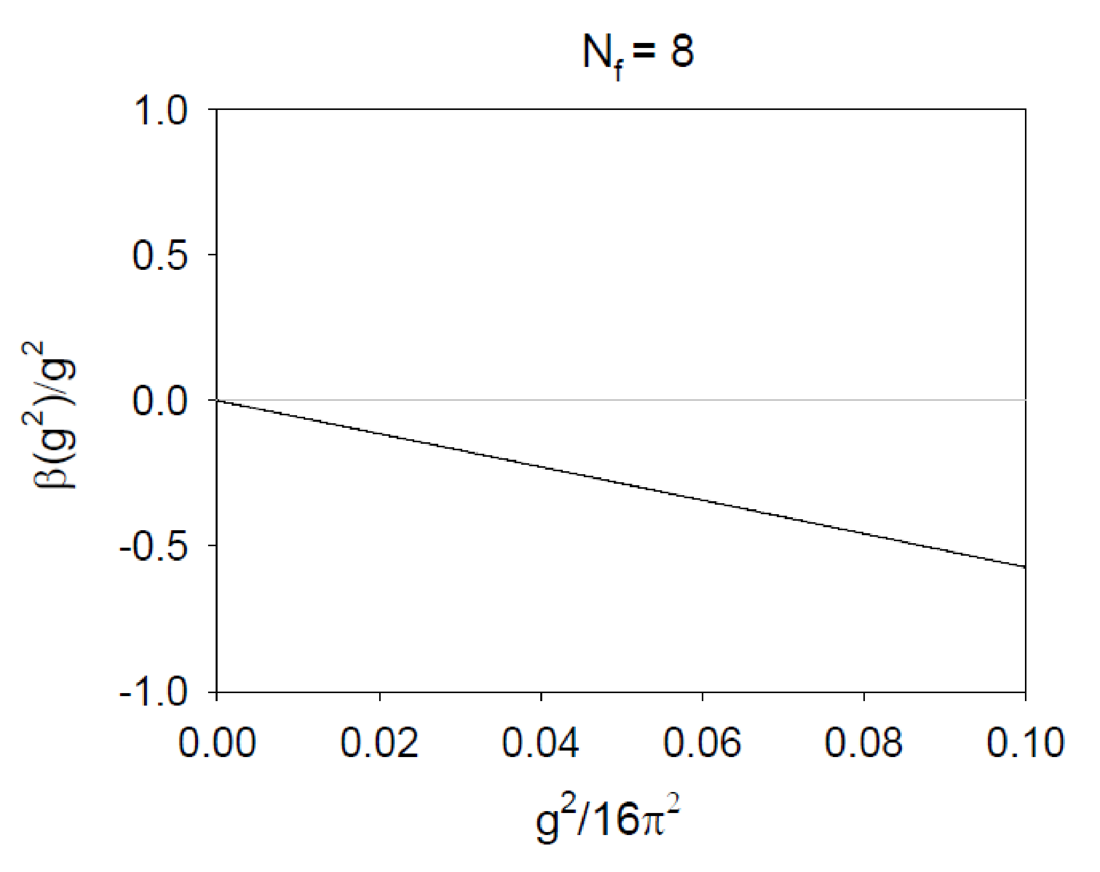

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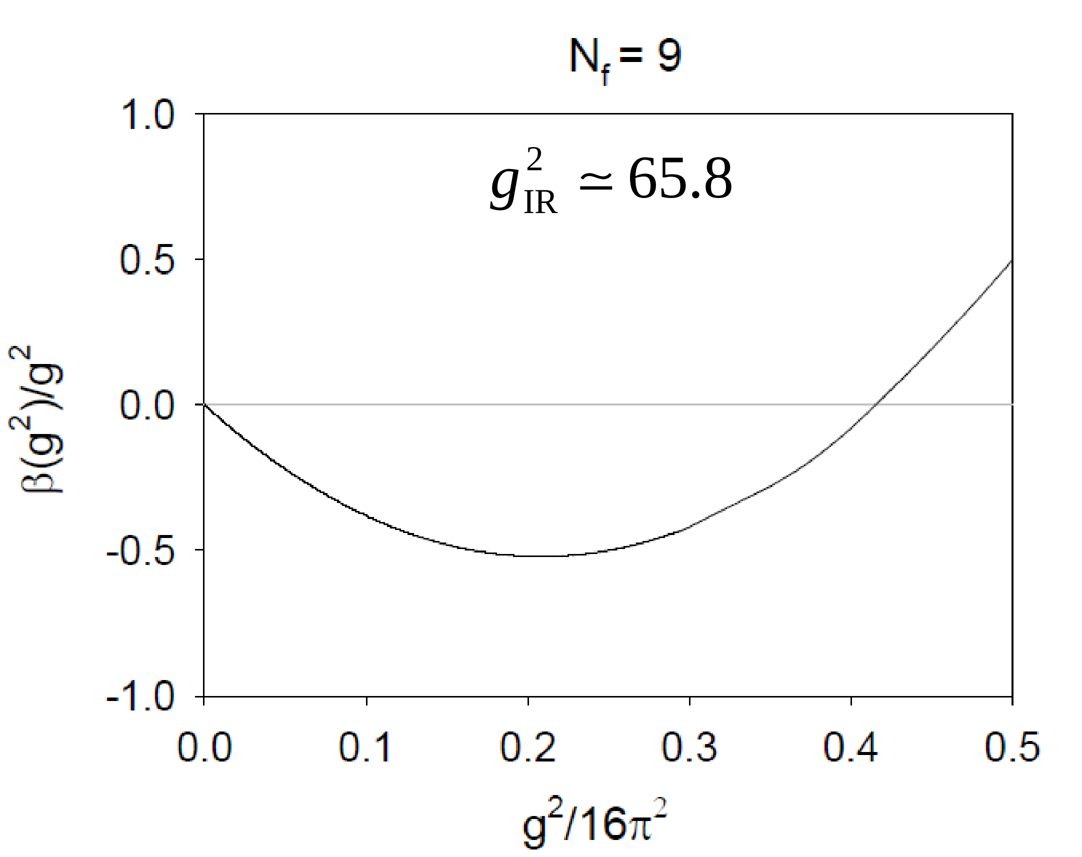

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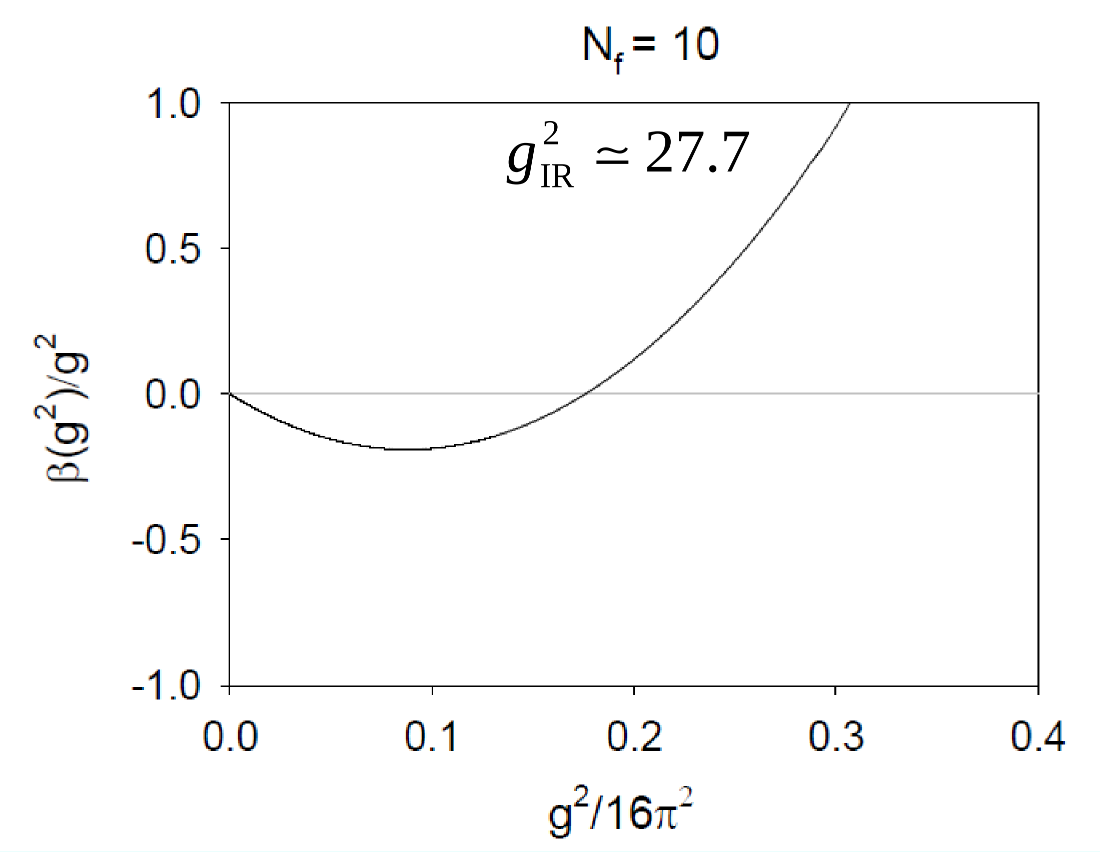

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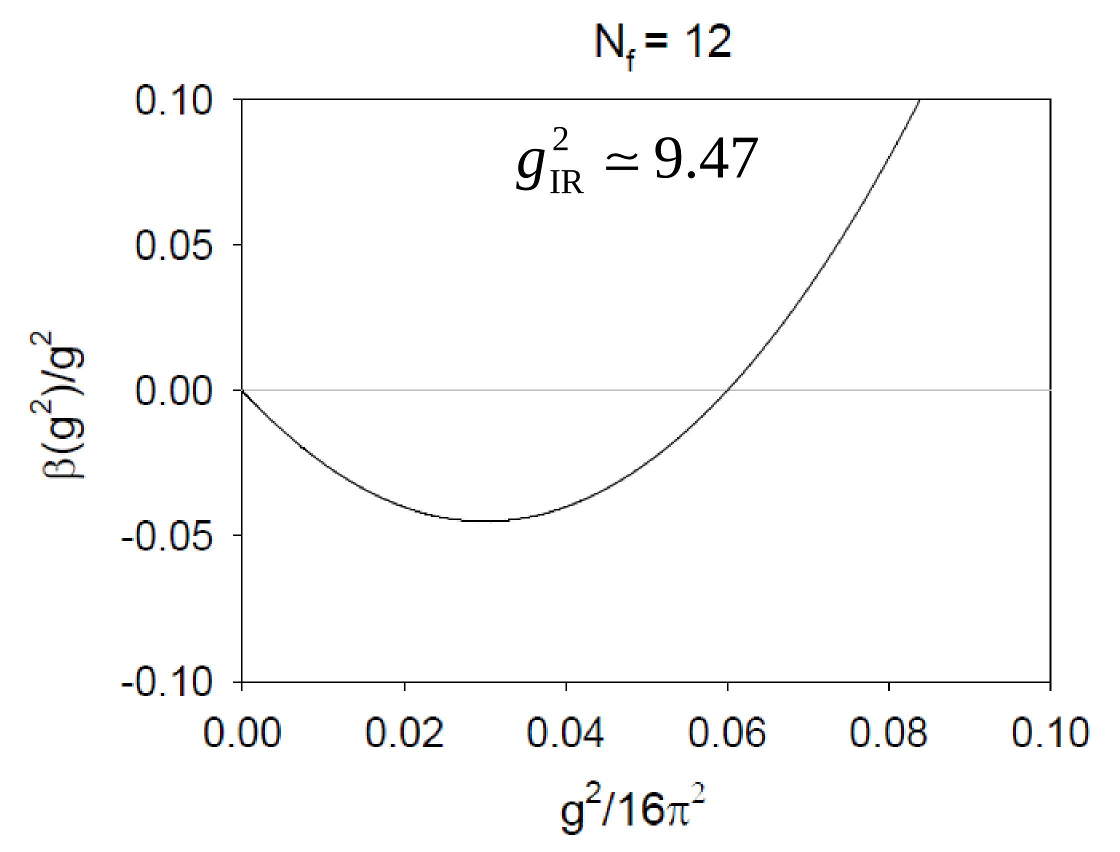

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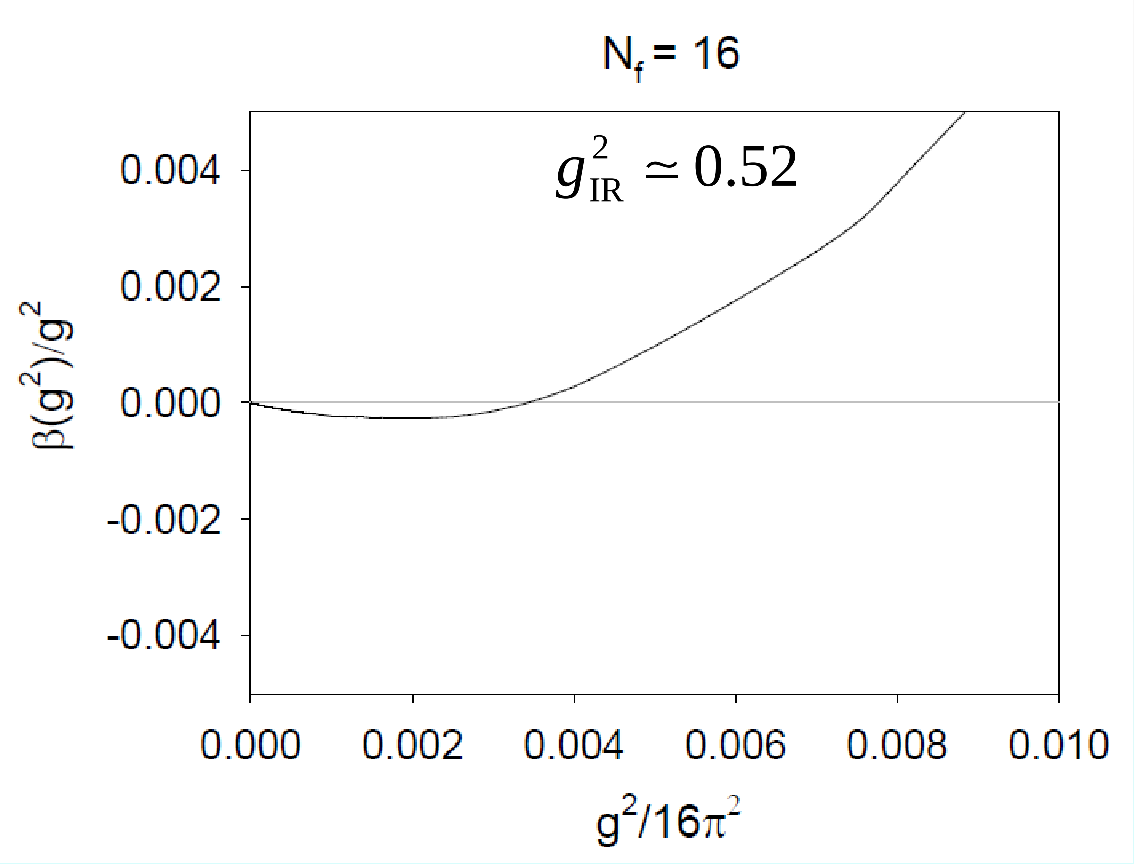

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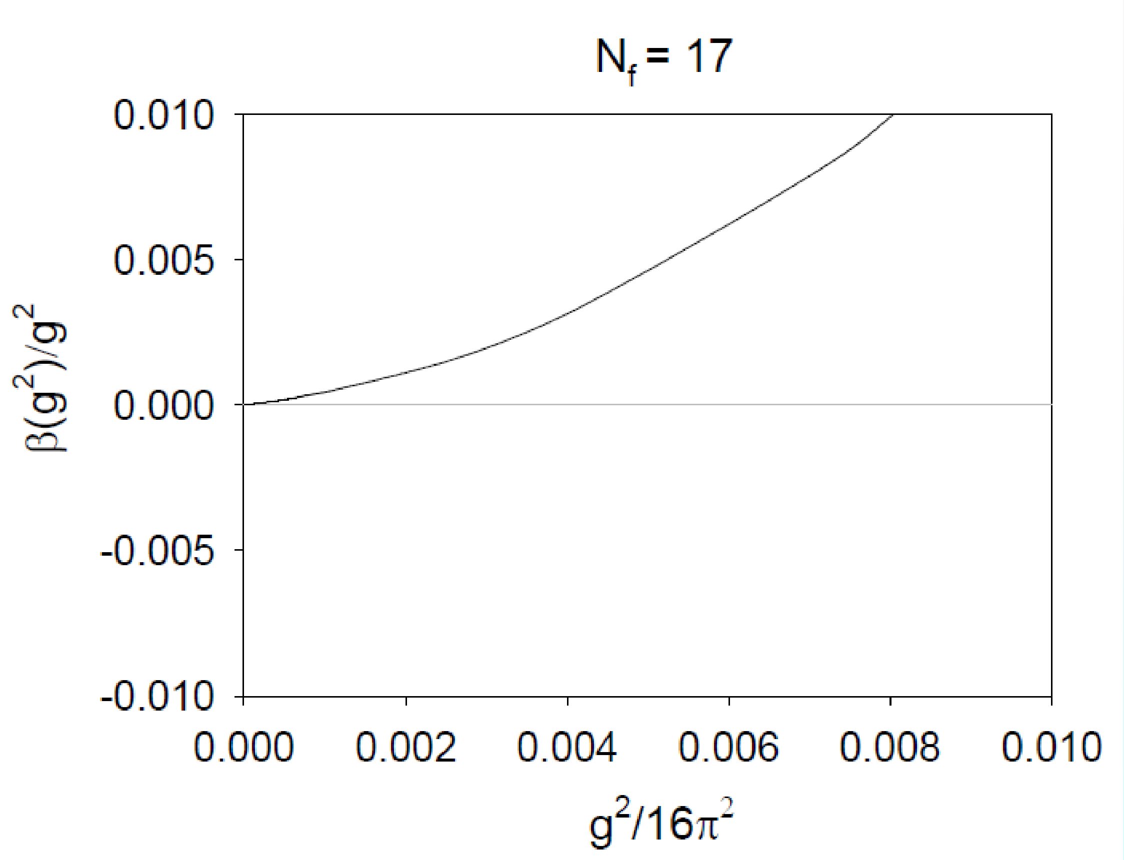

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- **The strong interaction physics around IRFP may have significant impacts to the phenomenological models beyond the Standard Model, e.g., models of composite Higgs boson.**
- **Lattice field theory provides a viable framework for nonperturbative study of vector gauge theories.**
- **It is vital to use lattice fermions with exact chiral symmetry , having exactly the same flavor symmetry as their counterparts in the continuum.**

Finite Volume Gradient Flow Scheme

The gradient flow (Wilson flow) amounts to solving

$$\frac{dB_\mu}{dt} = D_\nu G_{\nu\mu}, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\nu = \partial_\mu + [B_\mu, \ ],$$

in the flow time t with the initial condition $B_\mu\big|_{t=0} = A_\mu$

The gradient flow is a process to average the gauge field over a 4D spherical range with $R_{\text{rms}} = \sqrt{8t}$

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The renormalized coupling on the lattice L^4 is obtained from

$$g_c^2(L,a) \propto \left\langle t^2 E(t) \right\rangle, \quad c = \frac{\sqrt{8t}}{L} = \text{constant},$$

$$E(t) = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}(t), \ \text{energy density}$$

The proportional constant is fixed by requiring $g_c^2(L,a) = g^2_{\overline{MS}}$ to the 1-loop order.

Simulation of SU(3) gauge theory with Nf=10

- Gauge Action: Wilson plaquette action
  $\beta = 6/g_0^2$ = 6.45, 6.5, 6.6, 6.7, 6.8, 7.0, 7.5, 8.0, 9.0, 10.0, 12.0, 15.0

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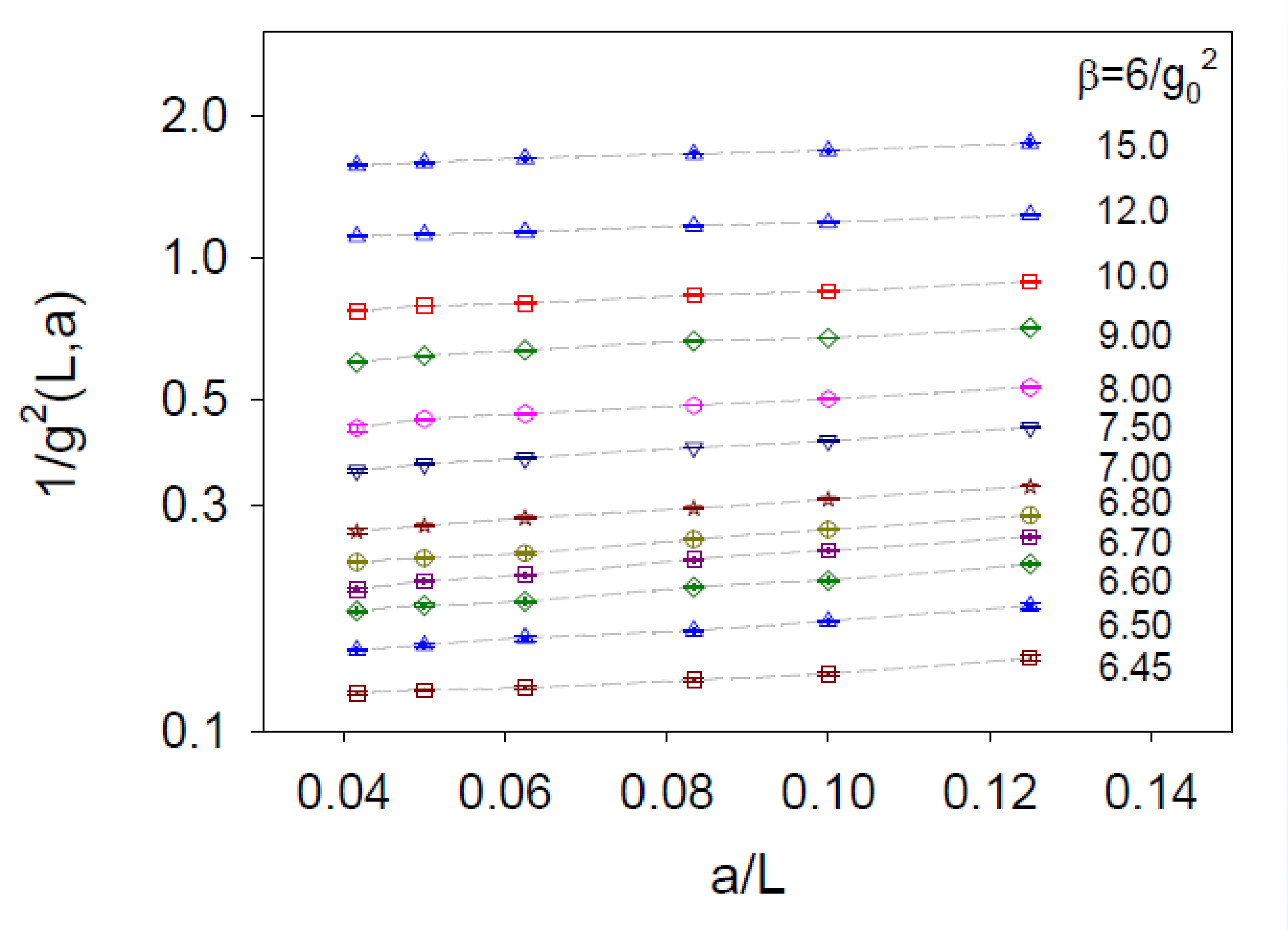
- The residual mass of any gauge ensemble is less than **5×10^{-5}**

Renormalized Couplings with $c = \frac{\sqrt{8t}}{L} = 0.3$



Other values of $g^2(L,a)$ are obtained by cubic spline interpolation.

Discrete β-function (DBF)

$$\beta(s, a/L) = \frac{g^2(sL,a) - g^2(L,a)}{\ln(s^2)}, \quad \text{for each pair of lattices } (L, sL).$$

Given a set of lattice pairs with a fixed ratio s, $\lim_{a \to 0} \beta(s, a/L)$ can be obtained by extrapolation. Moreover, if $\lim_{a \to 0} \beta(s, a/L)$ is available for several s, then the limit $s \to 1$ can be taken,

$$\lim_{s \to 1} \lim_{a \to 0} \beta(s, a/L) = -\beta(g^2) = -\frac{dg^2}{d \ln \mu^2}$$

If $\lim_{a \to 0} \beta(s, a/L)$ has IRFP, then $\beta(g^2)$ also has IRFP, and vice versa.

In the following, use lattice pairs $(8,16)$, $(10,20)$, $(12,24)$ with $s = 2$,

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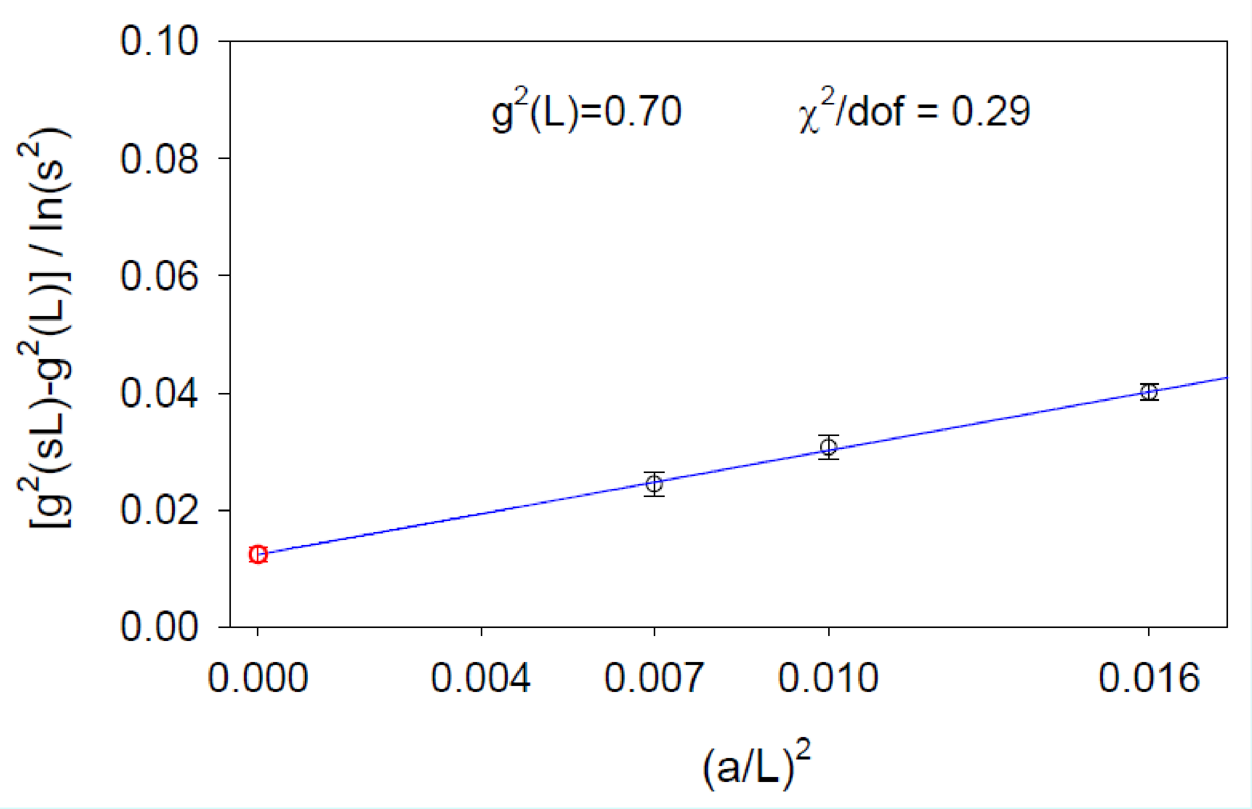

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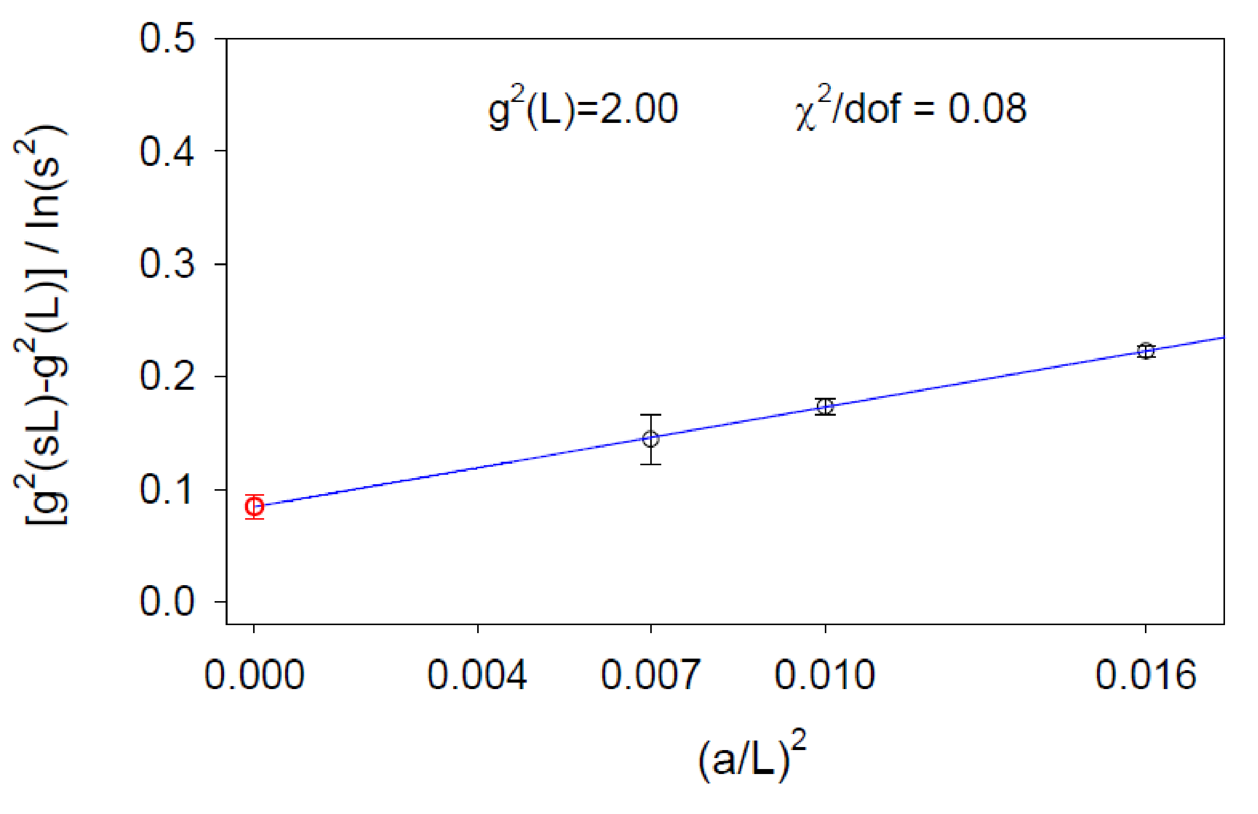

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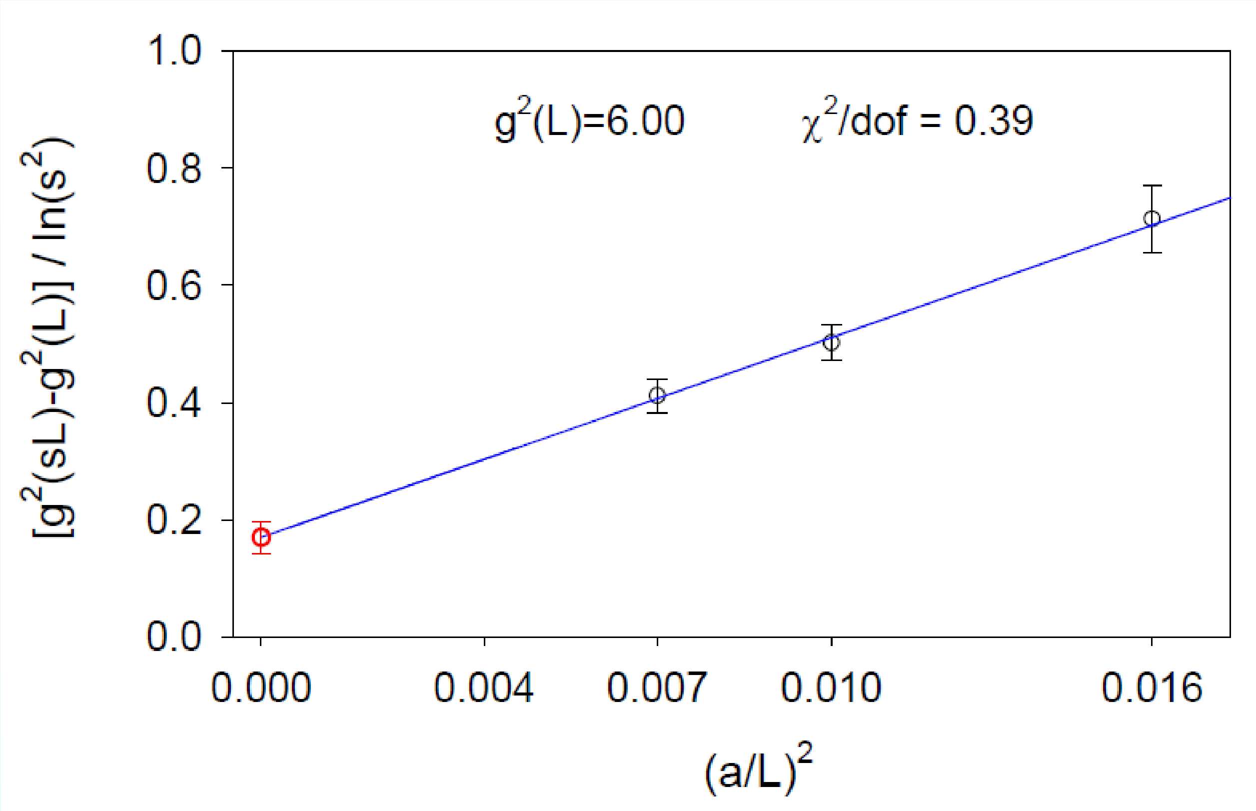

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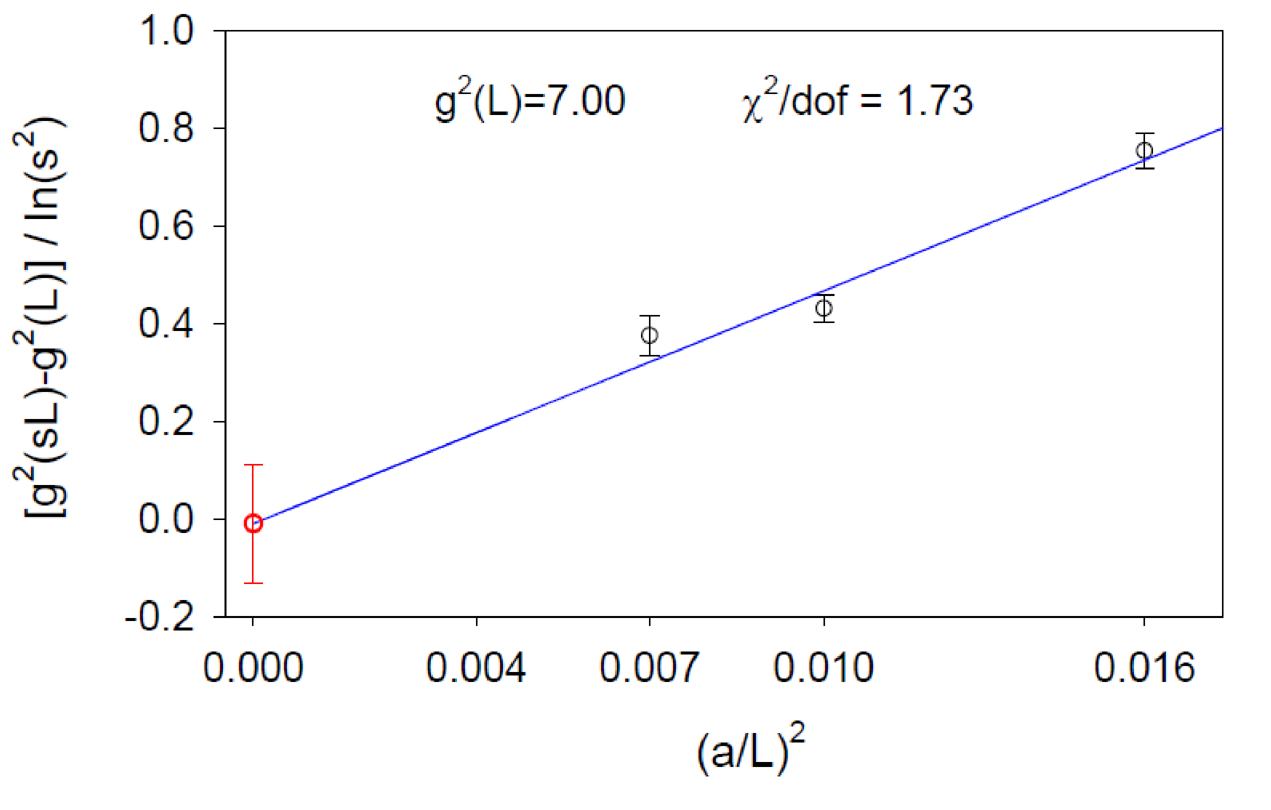

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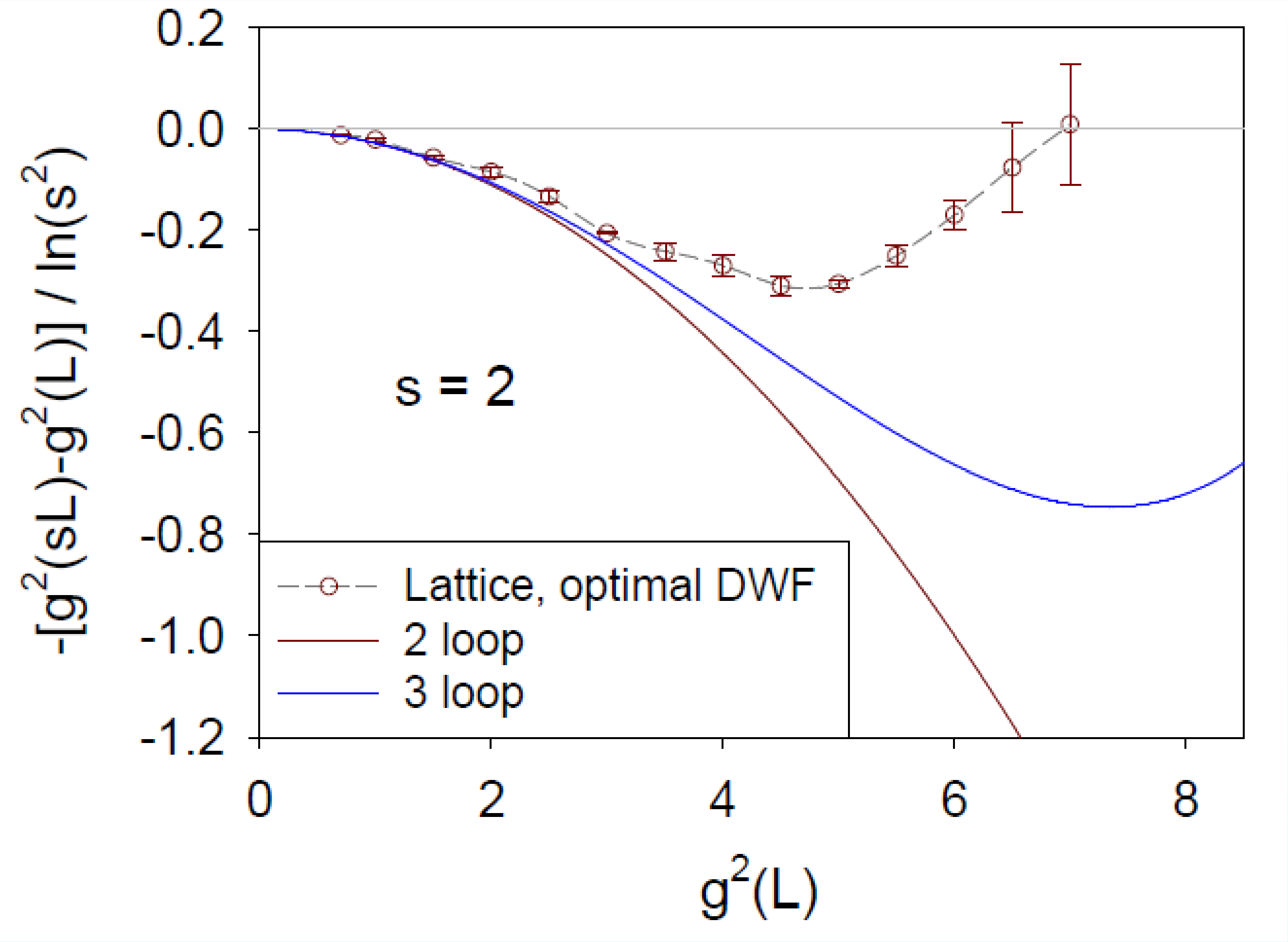

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- The present data of the discrete β-function (with $s=2$) of SU(3) gauge theory with 10 massless optimal DWF suggests that the theory may possess IRFP at $g_c^2 \sim 7.0$, and its long-distance behavior is conformal.

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- The statistical+systematic errors of the discrete β-function around the IRFP can be further reduced by increasing the statistics as well as more data points around the regime of strong couplings. Moreover, it is instructive to simulate L=32 (a difficult task !) and add the fourth lattice pair (16,32) to the existing data of DBF.