Sphaleron and Unsuppressed Baryon Number Violating Scatterings

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December 11, 2015
NCTS Annual Theory Meeting 2015:
Particles, Strings and Cosmology, NTHU, Hsinchu, Taiwan

Sphaleron and Baryon Number Violation

This talk is based on work done :

with **Sam Wong** : arXiv : 1505.03690
"Bloch Wave Function for the Periodic Sphaleron Potential and Unsuppressed Baryon and Lepton Number Violating Processes"

"Chern-Simons Number as a Dynamical Variable" (to appear)

with **Razei Emami and Sam Wong**
"Are Baryon Number Violating Processes Observable in the Laboratory ? (under preparation)

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Earlier, it was estimated that $(B + L)$-violating scattering processes will never be observed in the laboratory.

We re-examine this crucial point and argue otherwise

Anomaly in Electroweak Theory

- The SU(2) gauge theory is the cornerstone of the electroweak theory, in which the non-Abelian gauge fields couple to the Higgs field and the left-handed quarks and leptons.

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- A quantity that is conserved classically may not be conserved quantum mechanically.
- The violation of such conservation is due to "anomaly".
- Electric charge is always conserved. So are $(B - L)$ and energy-momentum.
- But not $(B + L)$, where B(proton)$= 1$ and L(electron)$= 1$.

$(B+L)$ violating Processes

- Example : 3 hydrogen atoms $\rightarrow \nu_e \nu_e \bar{\nu}_\mu \bar{\nu}_\tau$

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- This phenomenon may be understood in terms of sphaleron.

- Matter-anti-matter asymmetry in our universe may be due to the sphaleron in the electroweak theory in the early universe when the temperature is high.

  Kuzmin, Rubakov, Shaposhnikov, 1985, Shaposhnikov 1987, Fukugita and Yanagida 1988

  This possibility has been extensively studied.

Anomaly and Instanton in the Electroweak Theory

Keeping only the important parts in the electroweak theory,

$$\mathcal{L} = -\frac{1}{2} Tr[F_{\mu\nu} F^{\mu\nu}] + \frac{1}{2} \left(D_\mu \Phi\right)^\dagger D^\mu \Phi - \frac{\lambda}{4} \left(\Phi^\dagger \Phi - v^2\right)^2 + i \bar{\Psi}_L^{(i)} \gamma^\mu D_\mu \Psi_L^{(i)}$$

$$J_L^{(i)\mu} = \bar{\Psi}_L^{(i)} \gamma^\mu \Psi_L^{(i)}$$

$$\partial_\mu J_L^{(i)\mu} = \frac{g^2}{16\pi^2} Tr\left[F_{\mu\nu} \tilde{F}^{\mu\nu}\right], \qquad i = 1, 2, 3, ...12.$$

Belavin, Polyakov, Schwartz and Tyupkin (1975) constructed topological soliton solutions (instantons) in 4-dimensional Euclidean SU(2) theory:

$$N = \frac{g^2}{16\pi^2} \int d^4 x Tr\left[F_{\mu\nu} \tilde{F}^{\mu\nu}\right]$$

where the topological (Chern-Pontryagin) index N takes only integer values.

Vacuum Structure of non-Abelian (Yang-Mills) Gauge Theory

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In QCD, the actual vacuum state is the Bloch wave labelled by θ,

$$|\theta\rangle = \sum_k \exp(ik\theta)\,|k\rangle$$

Chern Character to Chern-Simons Number

Instead of going to Euclidean space, we can describe this phenomenon in our usual $3+1$ space-time.

Manton (1983) and Klinkhamer and Manton (1984) constructed the "sphaleron", i.e., the unstable extremum static solution in the SU(2) part of the electroweak theory.

We trade $A_\mu(\hat{t}, x, y, z) \to A_i(n, x, y, z)$ to get the potential with the Chern-Simons number n (or $n\pi$) as the coordinate

$$n = \frac{g^2}{16\pi^2} \int d^4x Tr \left[F_{\mu\nu} \tilde{F}^{\mu\nu} \right] = \int d^3x dt \partial_\mu K^\mu = \int d^3x K^0 \Big|_{t=t_0}$$

$$K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left(F^a_{\nu\rho} A^a_\sigma - \frac{g}{3} \epsilon^{abc} A^a_\nu A^b_\rho A^c_\sigma \right)$$

$(B+L)$-Violation

In electroweak theory,

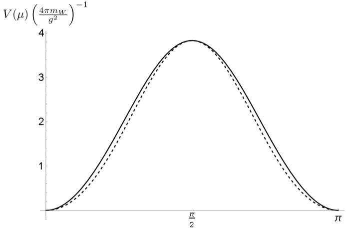
$$\Delta N_e = \Delta N_\mu = \Delta N_\tau = N$$

and the change in the baryon number B given by

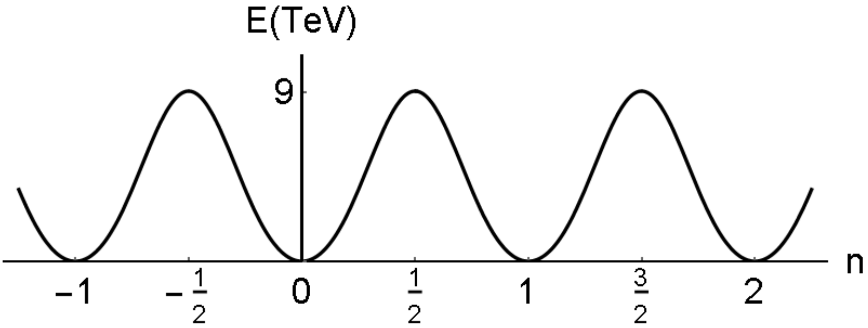
$$\Delta B = \frac{1}{3}(3)(3)N = 3N$$

where the $(B+L)$ number is violated. ('tHooft 1976)
Since the electric charge Q is always conserved, we have

$$\Delta(B+L) = 6N \qquad \Delta(B-L) = \Delta Q = 0$$



Periodic Sphaleron Potential



Height of potential = E(sphaleron) = 9.0 TeV and $n = \mu/\pi$.

We get the kinetic term for $\mu(t)$ via treating $\mu(t)$ (or $n(t)$) in $A_i(\mu(t), r, \theta, \phi)$ and $\Phi(\mu(t), r, \theta, \phi)$ as a function of time.

One-dimensional Schrödinger Equation

Mass m =17.1 TeV with coordinate $Q = \mu/m_W$, where $n\pi = \mu - \sin(2\mu)/2$,

$$L = \frac{1}{2}m\dot{Q}^2 - V(Q)$$

Next we quantize this system with Q (or n) as the coordinate to obtain the one-dimensional time-independent Schrödinger equation:

$$\left(-\frac{1}{2m}\frac{\partial^2}{\partial Q^2} + V(Q)\right)\Psi(Q) = E\Psi(Q)$$

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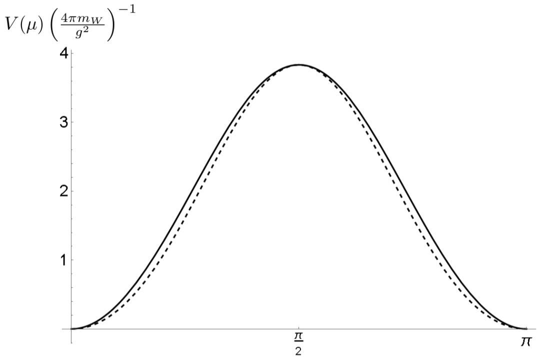
It is then straightforward to solve this using the Bloch Theorem to get the conducting (pass) band structure (one-dimensional Brillouin zone) :

Earlier Semi-classical Analysis

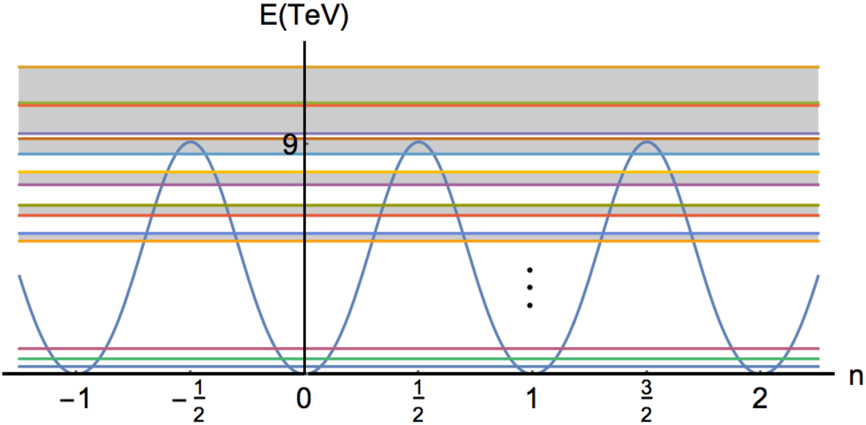


$$A \sim e^{-2\pi/\alpha_W} \sim e^{-\int dQ\sqrt{2mV_{sph}}} \sim e^{-\pi\sqrt{2mV_{sph}}/m_W}$$

$$V_{sph} \sim m_W/\alpha_W \to m \sim 2m_W/\alpha_W$$

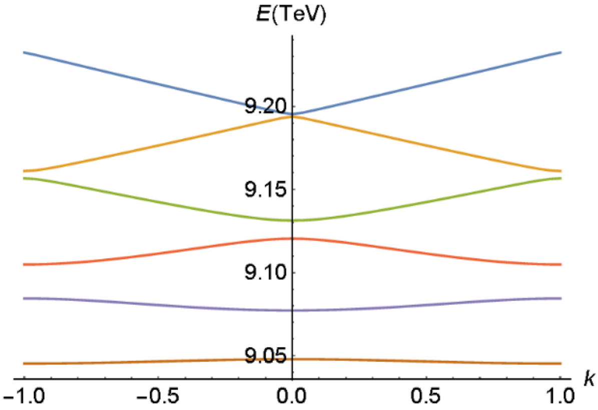
Earlier analysis never tried to find m or fully quantize Q.

Bloch Waves and Band Structure

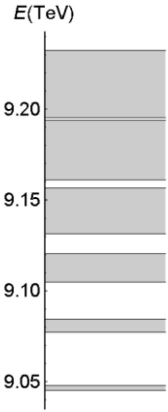


Here n is the Chern-Simons number.

Band Structure



(a)



(b)

$E_{sphaleron} = 9.11$ TeV

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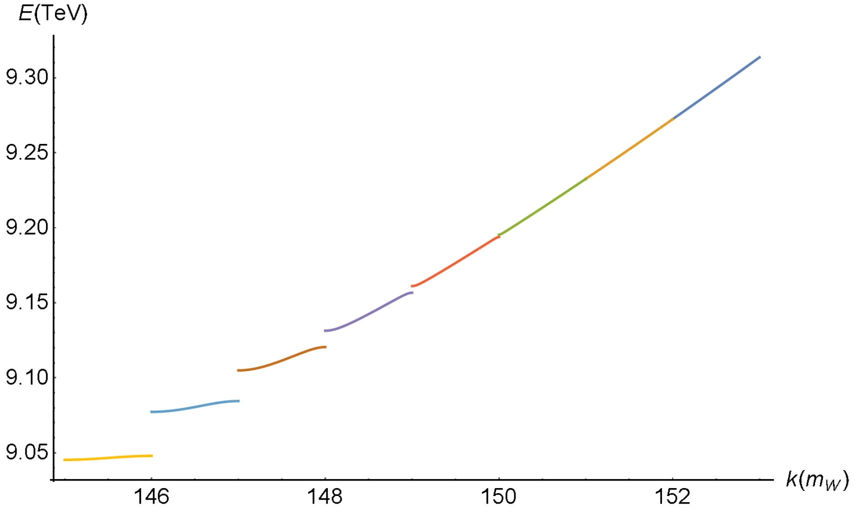
- If the wavefunction energy is spread over a GeV, then only 10^{-180} fraction would pass through the lowest band unsuppressed.
- This is how tunneling suppression is reproduced in this approach.
- The higher bands have larger band widths. For $E \geq E_{sphaleron}$,

$$\Psi(Q,t) \sim e^{ikQ - i\mathcal{E}t}$$

where $k^2/2m \sim \mathcal{E} = E - 4.1$ TeV.

Difference

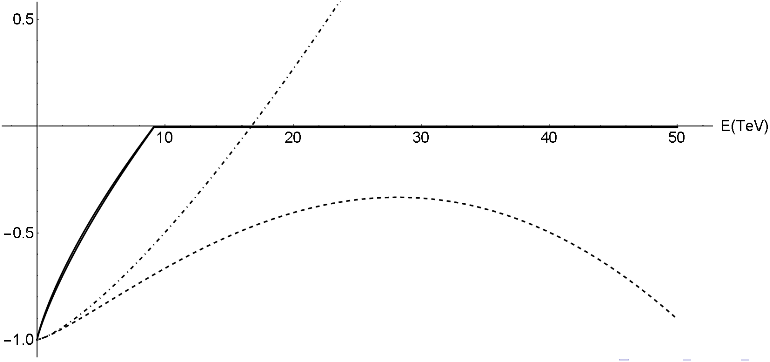


$$\mathcal{E} = E_{qq} - 4.1\,\text{TeV} = k^2/2m$$

$$\sigma_{\Delta n \neq 0}(2 \to \text{"}any\text{"}) \sim \exp\left(c(\frac{4\pi}{\alpha_W}) \left[-1 + \frac{9}{8}(\frac{E}{E_0})^{4/3} - \frac{9}{16}(\frac{E}{E_0})^2 + ... \right] \right)$$

where $E_0 = \sqrt{6}\pi m_W/\alpha_W = 15$ TeV and $c \sim 2$.

Earlier claim : Unobservable rate even at very high energies.



Phenomenology : LHC and Beyond

In the coming LHC run : $\Delta n = +1$ process:

$$u_L + d_L \to e^- \mu^- \tau^- bbtccsuuuud + ...$$

so a single pp (i.e., quark-quark) collision can produce 3 same sign charged leptons and 5 baryons (plus $B = L = 0$ content).

Similarly, one has $\Delta n = -1$ process:

$$q_L + q_L \to \bar{l}_e \bar{l}_\mu \bar{l}_\tau \bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q} + ...$$

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At higher energies, say at a 100 TeV collider, one can have $\Delta n = -1 - K$ processes

$$q_L + q_L \to \bar{l}_e \bar{l}_\mu \bar{l}_\tau \bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q} + [\bar{l}_e \bar{l}_\mu \bar{l}_\tau \bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}]^K + ...$$

Taking $f(14\text{TeV}, 9\text{TeV}) \sim 10^{-8}$ and an integrated luminosity of $L_{pp} = 3000\ fb^{-1}$ (inverse femtobarns), we guess the number of $(B+L)$-violating events with 3 same sign charged leptons to be:

$$
\begin{aligned}
&\text{Integrated event number in the LHC 14 TeV run} \\
&\sim \sigma(pp) \cdot f(14,9) \cdot F_{EW} \cdot \kappa \cdot F_3 \cdot L_{pp} \\
&\sim (80 \times 10^{-3} b)(10^{-8})(10^{-2})(10^{-2}/3)(\frac{1}{8})(3000 \times 10^{15} b^{-1}) \\
&\sim 10^4 \to 10^{4 \pm 2}
\end{aligned}
$$

where we naively take $F_{EW} \sim 10^{-2}$ and the fraction $\kappa \sim 10^{-2}/3$. Increasing the pp energy $E(pp)$ by just a few TeV above 14 TeV will increase the event rate by a few orders of magnitude.

$$\kappa(E) = \frac{\sigma(E, \Delta n = \pm 1)}{\sigma_{EW}(E)} \lesssim \frac{\sigma(E, \Delta n = \pm 1)}{\sigma_{EW}(E, \Delta n = 0)}$$

Earlier : Rubakov, Rebbi, Bezrukov, Ringwald, Espinosa, Shaposhnikov, Cohen, ...

$$\kappa < 10^{-60}$$

even for energy up to 100 TeV.

We have

$$\kappa \sim 10^{-3} \sim 10^{-2} \to 1$$

Here is an 60 order-of-magnitude difference in Higgs physics !!!!!

THANKS