Relaxion from N axions

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KC, S. H. Im, arXiv:1511.00132

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Outline

- Relaxion mechanism for the gauge hierarchy problem

  Big hierarchy between relaxion scales

- A mechanism to generate hierarchical relaxion scales

  (Exponentiated) clockwork mechanism

- UV completion with high scale SUSY

- Conclusion

Gauge hierarchy problem:

$$\mathcal{L}_{\rm higgs} = D_\mu H^\dagger D^\mu H - m_H^2|H|^2 - \frac{1}{4}\lambda|H|^4 + y_t H q_3 u_3^c + \ldots$$

$$\Rightarrow \quad \delta m_H^2 = \left[-3y_t^2 + 3\lambda + \frac{9g_2^2+3g_1^2}{8} + \ldots\right]\frac{\Lambda_{\rm SM}^2}{16\pi^2}$$

This requires a fine tuning if $\Lambda_{\rm SM}$ >> weak scale.

Possible solutions:

* New physics to regulate the quadratic divergence near the weak scale

  SUSY, composite Higgs, extra dim, …

* Multiverse

  Anthropic selection for m_H << $\Lambda_{\rm SM}$

* Cosmological relaxation

  Cosmological evolution of a scalar field (=relaxion) to select m_H << $\Lambda_{\rm SM}$

* N-naturalness, ….

Relaxion mechanism Graham, Kaplan, Rajendran '15

* Higgs boson mass is a dynamical field depending on the relaxion field ϕ.

$$m_H^2(\phi) = M^2 - g\phi + \ldots$$

$$\left(\text{This can be an approximation for } m_H^2(\phi) \, = \, M_1^2 + M_2^2 \cos\left(\frac{\phi}{f_{\rm eff}}\right) \text{ with a large } f_{\rm eff}.\right)$$

$M \sim \Lambda_{\rm SM}$ >> weak scale $v = 174 \text{ GeV}$

* Higgs boson mass is vanishing for certain value of ϕ:

$$m_H^2(\phi_0) \, = \, 0 \quad \text{at} \quad \phi_0 \, = \, M^2/g + \ldots$$

* Final Higgs boson mass (~ weak scale) is determined by the VEV of ϕ, which is stabilized at near ϕ_0 :

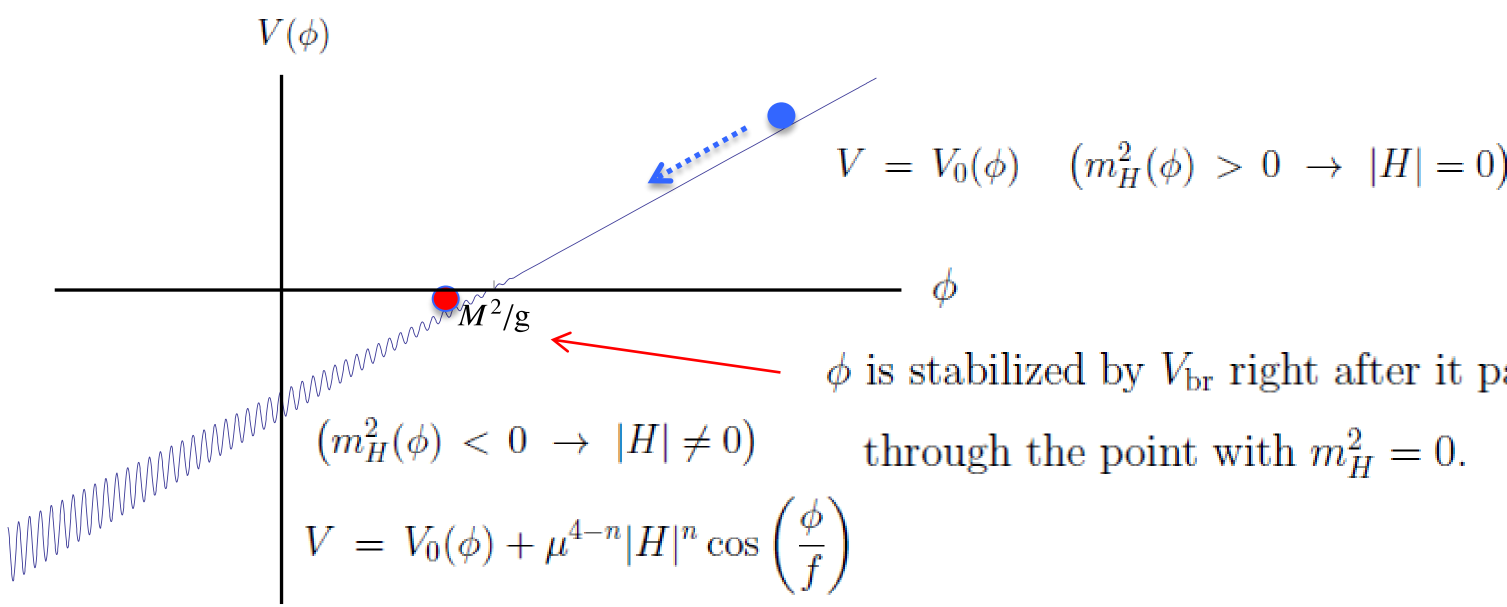
$$v \sim m_H(\langle\phi\rangle) \ll M \quad (\langle\phi\rangle \approx \phi_0)$$

Stabilizing ϕ at near ϕ_0 :

$$V(\phi, H) = V_0(\phi) + V_{\rm br}(\phi, H) + m_H^2(\phi)|H|^2 + \ldots$$

$V_0(\phi) = -gM_0^2\phi + \ldots \quad \left(V_0(\phi) = M_0^4 \cos\left(\frac{\phi}{f_{\rm eff}}\right) \text{ with a large } f_{\rm eff}.\right)$ enforcing ϕ to slide toward ϕ_0 during the period with $m_H^2(\phi) > 0 \rightarrow \langle H \rangle = 0$

$V_{\rm br} = \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f}\right)$ stabilizing ϕ near ϕ_0 when $m_H^2(\phi) < 0 \rightarrow \langle H \rangle \neq 0$



ϕ is stabilized by $V_{\rm br}$ right after it passes through the point with $m_H^2 = 0$.

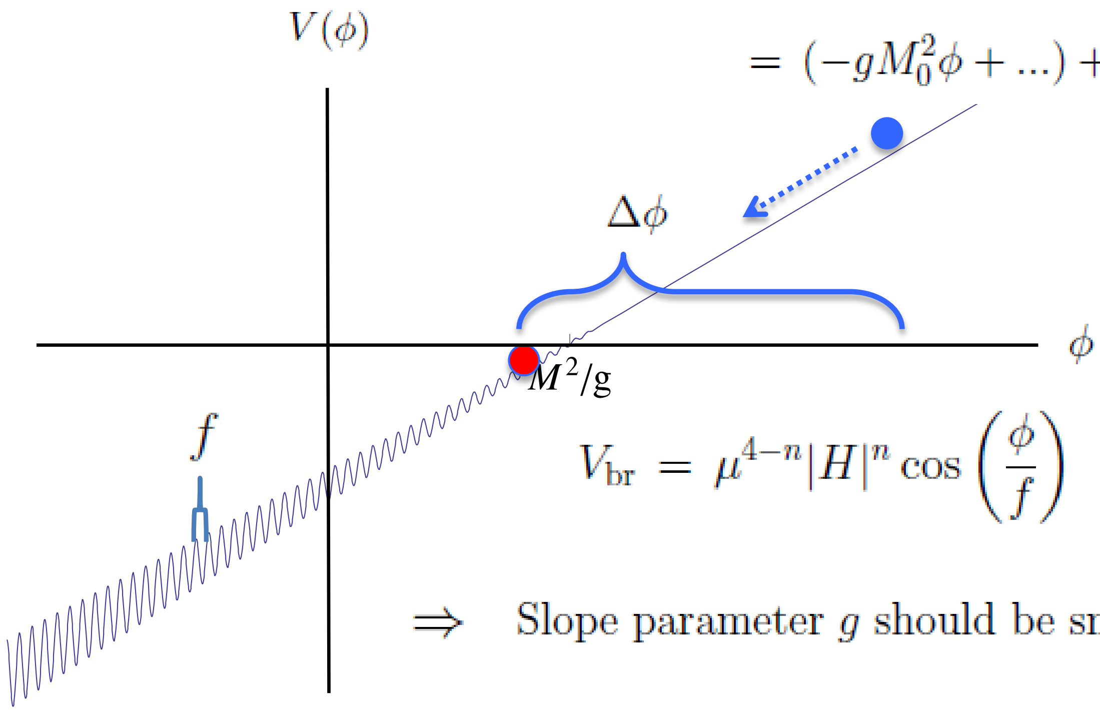
Naturalness conditions

* should be stable against power-law-divergent radiative corrections

* no fine tuning of the initial condition for the relaxion cosmology

$$V = V_0(\phi) + m_H^2(\phi)|H|^2 + \ldots$$
$$= (-gM_0^2\phi + \ldots) + (M^2 - g\phi + \ldots)|H|^2 + \ldots$$

$$(M_0 \sim M)$$



$$V_{\rm br} = \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f}\right) \lesssim v^4 \quad \text{at } |H| = v.$$

$\Rightarrow$ Slope parameter g should be small: $\dfrac{g}{f} \lesssim \dfrac{v^4}{M^2 f^2} \ll 1$

$\Rightarrow$ There should be a huge relaxion excursion to scan m_H^2 from $\mathcal{O}(M^2)$ to $\mathcal{O}(v^2)$:

Big hierarchy between the two relaxion scales: $\dfrac{f}{\Delta\phi} \lesssim \dfrac{v^4}{M^4} \ll 1$

Small back-reaction potential V_{br} and the resulting small slope parameter g are technically natural, i.e. stable against radiative corrections, as they correspond to tiny breaking of the shift symmetry:

$$\phi \to \phi + \text{constant}$$

Although technically natural, the required hierarchy in relaxion scales is so large, therefore calls for an explanation:

$$\frac{f}{f_{\rm eff}} \lesssim \frac{v^4}{M^4} \ll 1 \quad (f_{\rm eff} \sim \Delta\phi)$$

f = relaxion scale in $V_{\rm br}(\phi)$.

$f_{\rm eff}$ = relaxion scale in $V_0(\phi)$ and $m_H^2(\phi)$.

In some case, $\frac{f}{f_{\rm eff}} \lesssim 10^{-12}\left(\frac{v}{M}\right)^4$, so the scheme requires a much bigger hierarchy.

Also, typically the scheme requires a trans-Planckian value of $f_{\rm eff}$:

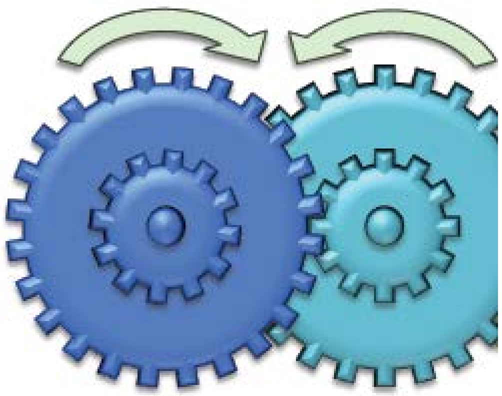
$$M \gtrsim 10^2\, v, \;\; f \gtrsim 10^8 \text{ GeV} \quad \to \quad f_{\rm eff} \gtrsim 10^{20} f \gtrsim 10^{10} M_{\rm Pl}$$

Q: What is the origin of this huge hierarchy between relaxion scales?

Clockwork mechanism for hierarchical axion scales

KC, Im, arXiv:1511.00132; Kaplan, Rattazzi, arXiv:1511.01827

$\frac{\phi_i}{f_i}$ = rotation of each wheel



$$V_{\rm cw} = -\Lambda^4 \cos\left(\frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2}\right)$$

$$\phi_H = \text{frozen heavy axion} \propto \frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2} = 0$$

$$\phi = \text{relaxion describing the collective rotation}$$

$$\Rightarrow \quad \frac{\phi_1}{f_1} = \frac{\phi}{f} \equiv n\frac{\phi}{f_{\rm eff}}, \quad \frac{\phi_2}{f_2} = -\frac{\phi}{f_{\rm eff}} \quad \left(f_{\rm eff} = \sqrt{n^2 f_1^2 + f_2^2} \equiv nf\right)$$

Recently a similar scheme has been applied for large field inflation:

KC, Kim, Yun, '14; Tye, Wong, '14; Ben-Dayan, Pedro, Westphal, '14; Harigaya, Ibe, '14; Bai, Stefanek, '15; de la Fuente, Saraswat, Sundrum, '15; ...

Clockwork mechanism for hierarchical axion scales

$$V = V_{\rm cw} + V_0 + V_{\rm br}$$

$$V_{\rm cw} = -\Lambda^4 \cos\left(\frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2}\right)$$

$$V_0 = -m_0^4 \cos\left(\frac{\phi_2}{f_2} + \delta_2\right)$$

$$V_{\rm br} = -\mu^{4-n}|H|^n \cos\left(\frac{\phi_1}{f_1} + \delta_1\right) \quad \text{with} \quad m_0^4,\ \mu^{4-n}v^n \ll \Lambda^4$$

$$\Rightarrow \quad \frac{\phi_1}{f_1} = \frac{\phi}{f} \equiv n\frac{\phi}{f_{\rm eff}}, \quad \frac{\phi_2}{f_2} = -\frac{\phi}{f_{\rm eff}} \quad \left(f_{\rm eff} = \sqrt{n^2 f_1^2 + f_2^2} \equiv nf\right)$$

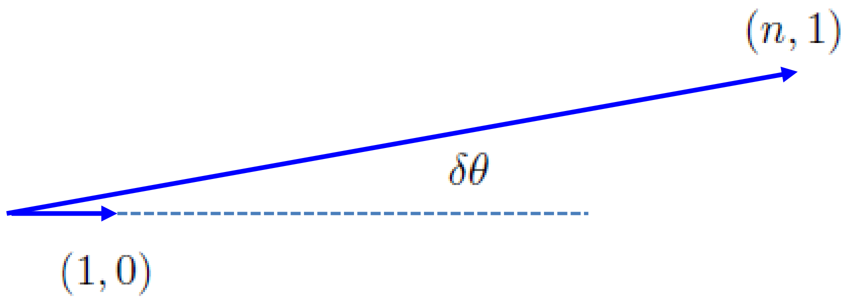
$$V_{\rm eff} = -m_0^4 \cos\left(\frac{\phi}{f_{\rm eff}} + \delta_2\right) - \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f} + \delta_1\right)$$

There are two axion scales in the effective theory, which are split by an integer n:

$$f_{\rm eff} = nf \sim nf_{1,2}$$

We need a big hierarchy: $\frac{f_{\rm eff}}{f} \gg 1$

* Alignment of two axion couplings: Kim, Nilles, Peloso '05



$$\frac{f_{\rm eff}}{f} = n \gg 1 \quad \left(\delta\theta \sim \frac{1}{n} \ll 1\right)$$

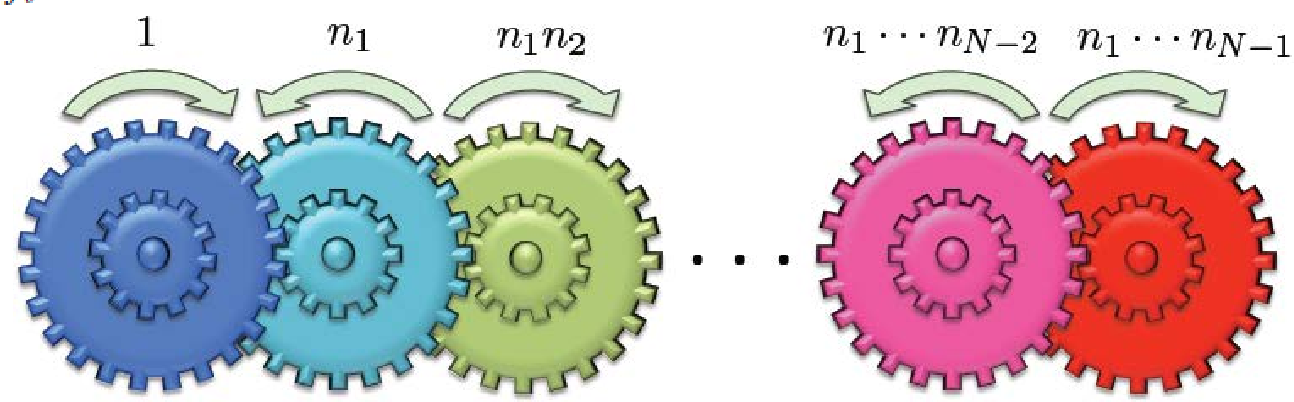
* Exponentiation with more (N > 2) axions: KC, Kim, Yun '14

$$\frac{f_{\rm eff}}{f} = n_1 n_2 ... n_{N-1} \sim e^N \quad \text{with} \quad N \gg 1$$

Exponentiation of the clockwork mechanism with N axions

KC, Im, arXiv:1511.00132; Kaplan, Rattazzi, arXiv:1511.01827

$\frac{\phi_i}{f_i}$ = rotation of each wheel



$$V_{\rm cw} = -\sum_{i=1}^{N-1} \Lambda_i^4 \cos\left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}}\right)$$

$$V_0 = -m_0^4 \cos\left(\frac{\phi_N}{f_N} + \delta_N\right)$$

$$m_H^2 = M_1^2 + M_2^2 \cos\left(\frac{\phi_N}{f_N} + \tilde{\delta}_N\right)$$

$$V_{\rm br} = \mu^{4-n}|H|^n \cos\left(\frac{\phi_1}{f_1} + \delta_1\right)$$

Exponentially many rotation of the last wheel

$\Rightarrow$ Exponentially long field space for the collective rotation

(= Relaxion ϕ with an exponentially large effective decay constant $f_{\rm eff} \sim n_1 n_2 ... n_{N-1} f$.)

Effective potential of the collective rotation angle (=relaxion):

$$\frac{\phi_1}{f_1} = \frac{\phi}{f}, \qquad \frac{\phi_N}{f_N} = (-1)^{N-1}\frac{\phi}{f_{\rm eff}}$$

$$\begin{aligned} V_{\rm eff} = & -m_0^4 \cos\left(\frac{\phi}{f_{\rm eff}} + (-1)^{N-1}\delta_N\right) \\ & + \left(M_1^2 + M_2^2 \cos\left(\frac{\phi}{f_{\rm eff}} + (-1)^{N-1}\tilde{\delta}_N\right)\right)|H|^2 \\ & - \mu^{4-n}|H|^n \cos\left(\frac{\phi}{f} + \delta_1\right) \end{aligned}$$

with an exponential hierarchy between axion scales:

$$f \sim f_i$$

$$f_{\rm eff} = \sqrt{\sum_{i=1}^{N}\left(\prod_{j=i}^{N-1} n_j^2\right) f_i^2} = n_1 n_2 \ldots n_{N-1} f \sim e^N f$$

Remark on the Weak Gravity Conjecture (WGC)

(Mild) WGC on the axion scale Arkani-Hamed et al, '06

There should exist an instanton which couples to the corresponding axion with a strength stronger than the gravity:

$$\mathcal{A}(\text{instanton}) \propto \exp\left(-S_{\text{ins}} + i\frac{\phi}{f}\right)$$

$$f \lesssim \frac{M_{\text{Pl}}}{S_{\text{ins}}}$$

Relaxion scheme involves an instanton generating $V_{\text{br}} \propto \cos\left(\frac{\phi}{f}\right)$ with

$$f \ll M_{\text{Pl}} \quad \left(f_{\text{eff}} \sim e^N f \gg M_{\text{Pl}}\right),$$

and therefore the scheme is apparently consistent with the WGC.

Brown, Cottrell, Shiu, Soler, '15

UV completed model with $M \sim \Lambda_{\rm SM} \sim m_{\rm SUSY}$

N axions from *N* U(1)'s:

$$U(1)_i : \quad X_i \to e^{i\beta_i} X_i, \quad Y_i \to e^{-3i\beta_i} Y_i \quad (i = 1, 2, ..., N)$$

X_i, Y_i : gauge-singlet chiral superfields

Murayama, Suzuki, Yanagida '92

Choi, Chun, Kim '96

$$W_1 = \sum_i \frac{X_i^3 Y_i}{M_*}$$

M_* : Cut-off scale such as the Planck or GUT scale

$$V(X_i, Y_i) = m_{X_i}^2 |X_i|^2 + m_{Y_i}^2 |Y_i|^2 + \left(A_i \frac{X_i^3 Y_i}{M_*} + \text{h.c} \right) + \frac{|X_i|^6}{M_*^2} + 9 \frac{|X_i|^4 |Y_i|^2}{M_*^2}$$

$$m_{X_i}^2 < 0, \; m_{Y_i}^2 > 0 \quad \to \quad X_i \sim Y_i \sim \sqrt{m_{\rm SUSY} M_*}$$

$$X_i \propto e^{i\phi_i/f_i}, \quad Y_i \propto e^{-3i\phi_i/f_i}$$

$$N \text{ axions } \phi_i \; (i = 1 \ldots N) \quad \text{with} \quad f_i \sim \sqrt{m_{\rm SUSY} M_*}$$

Hidden sector dynamics for exponentiated clockwork

$(V_{\rm cw}(\phi))$

- Introduce (N- 1) hidden Yang-Mills sectors.

Gauge group $G = \prod_{i=1}^{N-1} SU(p_i)$

Vector-like charged matter superfields $\Psi_i + \Psi_i^c, \quad \Phi_{ia} + \Phi_{ia}^c \quad (i = 1, 2, ..., N-1; a = 1, 2, ..., n_i)$

- The previous $U(1)_i$ charged field X_i couples to these matter fields through

$$W_2 = (X_1\Psi_1\Psi_1^c + X_2\Phi_{1a}\Phi_{1a}^c) + (X_2\Psi_2\Psi_2^c + X_3\Phi_{2a}\Phi_{2a}^c) + \cdots + (X_{N-1}\Psi_{N-1}\Psi_{N-1}^c + X_N\Phi_{N-1a}\Phi_{N-1a}^c)$$

$U(1)_{i,i+1} \times SU(p_i) \times SU(p_i)$ anomalies

Axion-dependent holomorphic gauge kinetic function of $SU(p_i)$

$$\tau_i = \frac{1}{g_i^2} + \frac{i}{8\pi^2}\left(\frac{\phi_i}{f_i} + n_i\frac{\phi_{i+1}}{f_{i+1}}\right) + \theta^2 M_{\lambda_i}$$

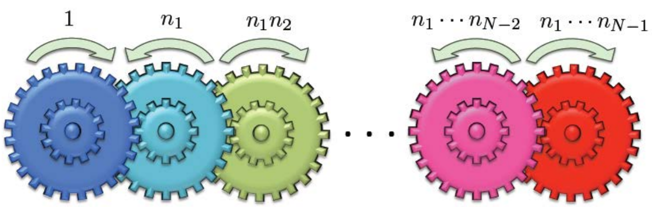
Hidden sector dynamics for exponentiated clockwork

$\left(V_{\rm cw}(\phi) \right)$

Confining scale of SU(p_i) $\Lambda_i \gg m_{\rm SUSY}$

gaugino condensation

$$W_{\rm np} \sim \langle \lambda_i \lambda_i \rangle \propto \left(\exp\left(-8\pi^2 \tau_i\right)\right)^{1/p_i}$$



$$V_{\rm cw} = \sum_{i=1}^{N-1} \frac{8\pi^2}{p_i} M_{\lambda_i} \Lambda_i^3 \cos\left(\frac{1}{p_i}\left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}}\right)\right)$$

Relaxion-dependent Higgs mass $(m_H^2(\phi))$

Superpotential to generate the MSSM μ-term

$$W_3 = \left(\frac{X_{N-1}^2}{M_*} + \frac{X_N^2}{M_*} \right) H_u H_d$$

Kim, Nilles '84

$$\mu_{\rm eff} = \mu_{N-1} \exp(-i2n_{N-1}\phi/f_{\rm eff}) + \mu_N \exp(i2\phi/f_{\rm eff})$$

$$B\mu_{\rm eff} = b_{N-1} \exp(-i2n_{N-1}\phi/f_{\rm eff}) + b_N \exp(i2\phi/f_{\rm eff})$$

$$f_{\rm eff} \sim n_1 n_2 \ldots n_{N-1} f_1 \sim e^N f_1$$

where $|\mu_N| \sim |\mu_{N-1}| \sim \frac{f^2}{M_*} \sim m_{\rm SUSY}, \quad |b_N| \sim |b_{N-1}| \sim m_{\rm SUSY}^2$

Naturally $\mu_{\rm eff} \sim m_{\rm SUSY}$

$$|\mu_{\rm eff}|^2 = |\mu_N|^2 + |\mu_{N-1}|^2 + 2|\mu_N \mu_{N-1}| \cos\left(2(n_{N-1}+1)\frac{\phi}{f_{\rm eff}} + \delta_{\mu_N} - \delta_{\mu_{N-1}} \right)$$

$$|B\mu_{\rm eff}|^2 = |b_N|^2 + |b_{N-1}|^2 + 2|b_N b_{N-1}| \cos\left(2(n_{N-1}+1)\frac{\phi}{f_{\rm eff}} + \delta_{b_N} - \delta_{b_{N-1}} \right)$$

The determinant of the Higgs mass matrix can be vanishing for certain value of the relaxion field.

Sliding relaxion potential ($V_0(\phi)$)

$$W_3 = \left(\frac{X_{N-1}^2}{M_*} + \frac{X_N^2}{M_*}\right) H_u H_d$$

radiative corrections

$$\Delta K = \frac{X_{N-1}^2 X_N^{*2}}{M_*^2} + \text{h.c.}$$

$$V_0 = -m_0^4 \cos\left(2(n_{N-1}+1)\frac{\phi}{f_{\text{eff}}} + \delta\right)$$

$$m_0^4 \sim \frac{f_{N-1}^2 f_N^2}{M_*^2} m_{\text{SUSY}}^2 \sim m_{\text{SUSY}}^4$$

Back reaction potential $(V_{\rm br}(\phi))$

Graham, Kaplan, Rajendran, '15

Option A: QCD

$$W_{\rm br} = X_1 Q Q^c \quad \rightarrow \quad \frac{1}{32\pi^2}\frac{\phi_1}{f_1} G\tilde{G}$$

$$\Rightarrow \quad V_{\rm br} \sim y_u \Lambda_{\rm QCD}^3 |H| \cos\left(2\frac{\phi}{f} + \delta_{\rm br}\right) \qquad f = \frac{f_{\rm eff}}{\left(\prod_{i=1}^{N-1} n_i\right)} \sim f_1$$

Option B: Hidden color dynamics

$$W_{\rm br} = \kappa_1 \frac{X_1^2}{M_*} L L^c + \kappa_u H_u L N^c + \kappa_d H_d L^c N$$

$\mathrm{SU(2)_L}$ doublet: $L + L^c$

$\mathrm{SU(2)_L \times U(1)_Y}$ singlet: $N + N^c$

Vector-like hidden colored matter fields

Integrating out L + L[c]

$$V_{\rm br} \sim \frac{\kappa_u \kappa_d \sin 2\beta}{m_L} \Lambda_{\rm HC}^3 |H|^2 \cos\left(2\frac{\phi}{f} + \delta_{\rm br}\right)$$

$$\Lambda_{\rm HC} < m_L \sim \kappa_1 m_{\rm SUSY} < 4\pi v$$

to keep the naturalness

Conclusion

- The relaxion mechanism offers an alternative solution to the gauge hierarchy problem, but it requires a huge relaxion excursion (typically trans-Planckian), leading to a big hierarchy between relaxion scales.

- We propose a scheme (=exponentiated clockwork) to generate an exponential hierarchy between axion scales within an EFT of multiple axions where all fundamental axion scales are well below the Planck scale.

- Our scheme finds a natural UV completion in high scale SUSY scenario.

- To complete the relaxion scheme, one needs to accommodate baryogenesis, inflation & dark matter in the scheme.