Superconformal Field Theories in Higher Dimensions

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Outline

- QFTs & SCFTs
- 6d superconformal field theories
- 6d little string theories
- 5d superconformal field theories
- Conclusion

Quantum Field Theory

- The theory of identical particles (free and weakly interacting particles)
- Poincare symmetry (space-time translation and Lorentz transformation), fermions and bosons
- gauge symmetry, chiral symmetry
- interaction, perturbative expansion, renormalization and RG flow
- nonabelian gauge symmetry, asymptotic freedom and confinement
- Goldstone bosons and Higgs mechanism,
- the standard model for elementary particles interacting with electroweak and strong forces

Conformal Field Theory (CFT)

- $d>2$: Poincare (P_μ,$M_{\mu\nu}$)+Scaling (D) + Special Conformal (K_μ)
- all massless, no scale
- conformal symmetry at IR or UV
- no concept of particles, or S-matrix for IR interacting conformal theory except when it is near the free theory
- tools to explore?
- gauge invariant local operators : $\phi(x)$ (local disturbance)
- correlation functions

CFT

mass dimension

- conformal dimension Δ: under $x^\mu \to \lambda x^\mu,\ \phi'(x) = \lambda^\Delta \phi(\lambda x)$
- unitary cft: for scalar, $\Delta \geq (d-2)/2$
- primary field (lowest dim for a representation): $[K_\mu, \phi(x)]=0$ for $x=0$

$$
\begin{aligned}
[P_\mu, \Phi(x)] &= i\partial_\mu \Phi(x), \\
[M_{\mu\nu}, \Phi(x)] &= [i(x_\mu \partial_\nu - x_\nu \partial_\mu) + \Sigma_{\mu\nu}]\Phi(x), \\
[D, \Phi(x)] &= i(-\Delta + x^\mu \partial_\mu)\Phi(x), \\
[K_\mu, \Phi(x)] &= [i(x^2\partial_\mu - 2x_\mu x^\nu \partial_\nu + 2x_\mu \Delta) - 2x^\nu \Sigma_{\mu\nu}]\Phi(x),
\end{aligned}
$$

- correlation functions: two, three

$$\langle \phi(0)\phi(x)\rangle \propto \frac{1}{|x|^{2\Delta}} \equiv \frac{1}{(x^2)^\Delta}$$

$$\langle \phi_i(x_1)\phi_j(x_2)\phi_k(x_3)\rangle = \frac{c_{ijk}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$$

Super Conformal Field Theory (SCFT)

- Poincare symmetry+ Fermionic charge Q :supersymmetric field theory: $[P_\mu]=1$, **[Q]=1/2**, {Q,Q}~P

- Warner Nahm's classification of superconformal algebra: $d \leq 6$

- [D]=[M]=0, $[K_\mu]=-1$, [S]=-1/2, [R]=0

$$[D,Q] = -\frac{i}{2}Q; \qquad [D,S] = \frac{i}{2}S; \qquad [K,Q] \simeq S; \qquad [P,S] \simeq Q;$$
$$\{Q,Q\} \simeq P; \qquad \{S,S\} \simeq K; \qquad \{Q,S\} \simeq M + D + R.$$

- maximally supersymmetric (scft) 32 : d=3,4,**6**

- 16 supersymmetric (scft): d=3,4,**5,6**

- 8 supersymmetric (scft): d=3,4

SCFT

- chiral primary operators: for some S, $[S,\phi(x)]=0$
- the dimension of a chiral operator is fixed by its Lorentz and R-symmetry charge $\{Q,S\}\sim M+D+R$
- counting chiral primary fields: radial quantization and index function on S^1xS^{d-1}
- DLCQ (discrete light cone quantization) index, vacuum partition function on S^d
- OPE and conformal bootstrap (4-point correlation functions)
- anomaly, ads/cft correspondence, entanglement entropy,…

SCFT in 4d

- N=4, Maximally supersymmetric Yang-Mills theory of gauge group G
- N=2 supersymmetric SU(N) theory with $N_f=2N$ fundamental hypermultiplets
- N=2 supersymmetric rank 1 theories, non-Lagrangian theories
  - Argyles-Douglas type, E_n-type
- N=1 supersymmetric theories with Seiberg-duality
- Partition functions, S-duality,…

6d SCFTs

- N=(2,0) SCFTs: ADE (nonabelian tensor multiplet)
  - A_N on N M5 branes, D_N on N M5+OM5 branes
  - type IIB on ADE singularity
- N=(1,0) SCFTs: quiver with multiple nonabelian YMs + tensor multiplets + hypermultiplets
  - M5 branes on Γ_{ADE} singularity
  - M5 branes near E_8 M9 wall
- 6d SCFTs → 5,4,3,2d QFTs

6d (2,0) SCFTs

- superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times Sp(2)_R$
  - fields: B, Φ_I, Ψ
  - selfdual strength $H=dB={}^*H$, purely quantum $\hbar=1$
  - superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times Sp(2)_R$
- tensor branch: M2 branes connecting M5 branes = selfdual
- We do not know how to write down the theory for nonabelian case.
- N^3 degrees of freedom

6d ADE (2,0) SCFTs

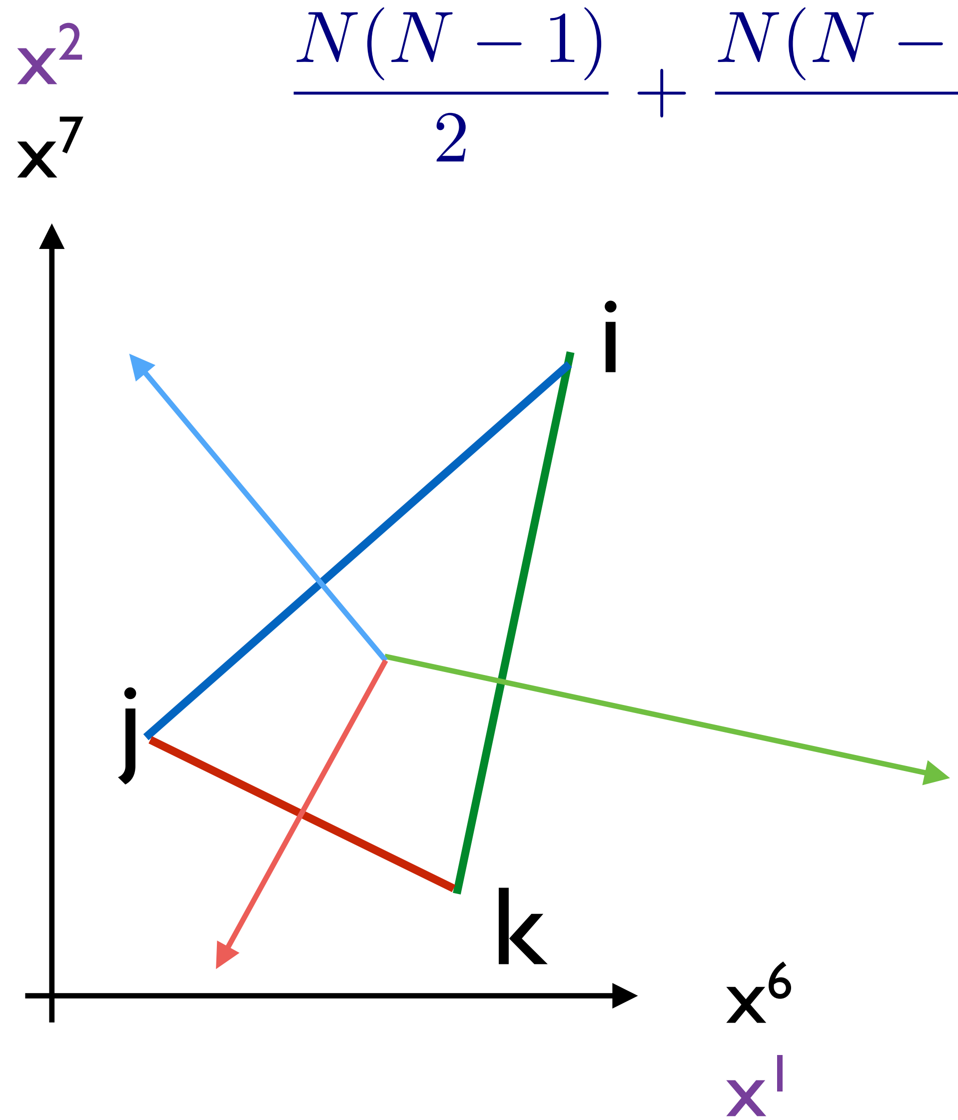
- Circle Compactification: 5d N=2 Super Yang-Mills $1/R = 8\pi^2/g_{YM}^2$
  - Instantons =Kaluza-Klein mode, no need for additional degrees of freedom
  - dyonic instanton index function (HC Kim,S Kim,E Koh,KL,S Lee 2011)
  - Complete by its own? (6-loop divergence on planar diagram) (Bern,Carrasco,Dixon,Douglas,Johansson,von Hippel)
- 5d Yang-Mills theory: Yang-Mills coupling in Coulomb phase
  - photon = massless mode on M5 branes $(H,\Phi_I,\Psi)=(3,5,8)$
  - W-bosons= selfdual strings wrapping the circle (α=i-j)
  - structure constants : $f^{\alpha\beta\gamma}$ with $\alpha+\beta+\gamma=0$, $f^{\alpha-\alpha h}$,
  - off-shell of 1/4 BPS: selfdual string with wave (i-j+ i or j)
  - off-shell of 1/4 BPS self-dual junctions (i,j,k or α=i-j,β=j-k,γ=k-i)

KL,Yee:0606150
Bolognesi,KL:1105.5073

1/4 BPS String Junctions

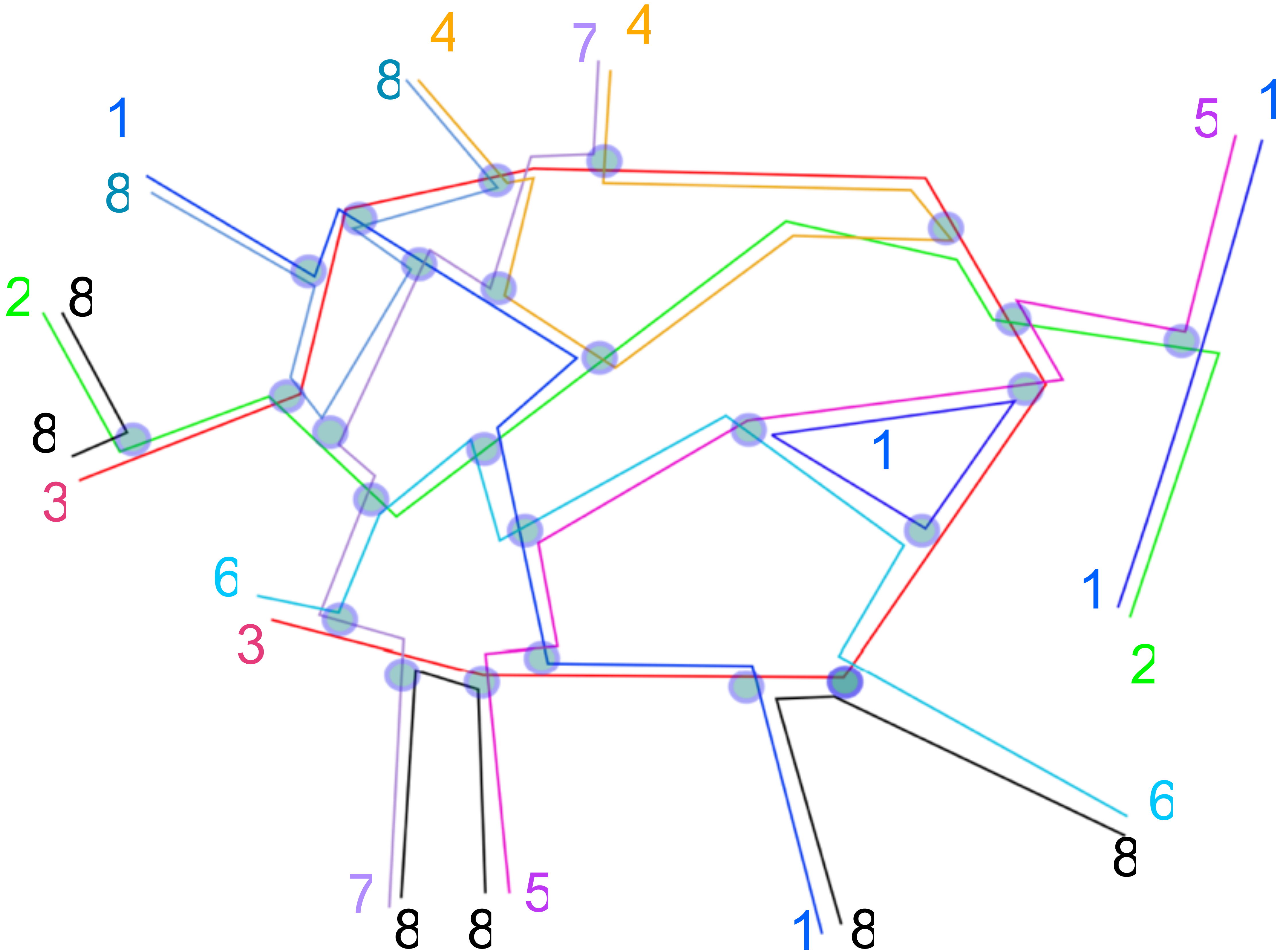
- fabc:,
  - $f^{\alpha\text{-}\alpha h}$ selfdual string with wave (α=i-j, h= i or j)
  - $f^{\alpha\beta\gamma}$ with α+β+γ=0, (i,j,k or α=i-j,β=j-k,γ=k-i)

$$\frac{N(N-1)}{2}+\frac{N(N-1)(N-2)}{6}=\frac{N(N^2-1)}{6}=h_G d_G$$



KL,Yee:0606150
Bolognesi,KL:1105.5073

high temperature phase



6d (2,0) SCFT on $S^1 \times S^5$

$$Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}, S = Q^\dagger$$

- No Lagrangian
- S^5 = CP^2 with S^1 fiber:

$$ds^2_{S^5} = ds^2_{CP^2} + (d\chi + V)^2, \; dV = 2J$$

- twisting: KK momentum

$$k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + \frac{p}{2}(R_1 - R_2), \; p = \text{odd integer}$$

- 5d Yang-Mills Chern-Simons theory
- p=-1/2 : $k = j_1+j_2+j_3+ R_1+ 2R_2$: 8 supersymmetries

$$\begin{aligned} S \; = \; & \frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\sqrt{|g|}\,\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta - \frac{2i}{3}A_\rho A_\sigma A_\eta\Big) \\ & -\frac{1}{2}D_\mu\phi_I D^\mu\phi_I + \frac{1}{4}[\phi_I,\phi_J]^2 - 2\phi_I^2 - \frac{1}{2}(M_{IJ}\phi_J)^2 - i(3-p)[\phi_1,\phi_2]\phi_3 - i(3+p)[\phi_4,\phi_5]\phi_3 \\ & -\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda - \frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda] - \frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn} + \frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \end{aligned} \qquad (2.27)$$

Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM to [arXiv:1307:7660]

6d (2,0) SCFT on $S^1 \times CP^2$

- ‘tHooft coupling: $\lambda = N/K$
- Partition function = 6d index function
- Vacuum of SU(N): $F = 2J(N-1, N-3, \ldots, -(N-1))$ = 2j x Weyl vector
- Vacuum energy: $-N(N^2-1)/6$
- Index matches with AdS/CFT calculation

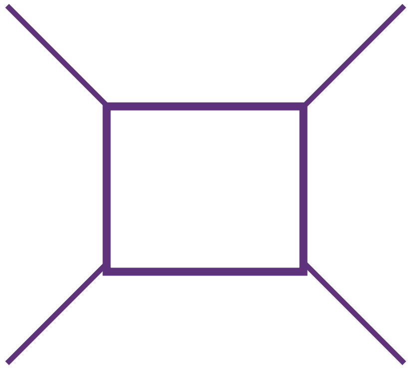
Hee-Cheol Kim, KM [arXiv:1210.0853]
Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM to [arXiv:1307:7660]

6d N=2 Little String Theory

- type IIB LST: On N NS5 branes with l_s fixed, g_s=0 limit
  - low energy 6d (1,1) YM with $g_2=l_s^2$
  - instanton string=little strings
- type IIA LST: On N NS5 branes with with l_s fixed, g_s=0 limit
  - low energy 6d (2,0) SCFT
- T-dual to each other on a circle compactification
  - winding-momentum exchange
- Elliptic genus calculation confirm T-duality: J. Kim, S. Kim, KL 2015

6d (1,0) SCFTs

- tensor branch
  - (1,0) vector multiplet: gaugeno (1,0)
  - (1,0) hypermultiplet: higgsino (0,1)
  - tensor multiplet: (0,1)
- gauge anomaly due to vector and matter 1-loop



- anomaly polynomial

$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_R = 0 \ for \ SU(2), SU(3), G_2, SO(8), F_4, E_6, E_7, E_8$$

$$c_{\mathrm{tot}} = \left[c_{Ad} - \sum_{R \ \mathrm{matter}} c_R \right] \geq 0$$

SU(n)+ tensor + matter

R	α_R	c_R
fund	1	0
asym	n-8	3
sym	n+8	3
adj	2n	6

- vector + 2n fund + tensor
  - two NS6 on n D6: two M5 on A_{n-1} singularity
- vector + 8 fund+ 1 asym+ tensor
  - O8+8D8 with n D6 connecting 1/2 NS brane on O8 and NS5 brane
- vector + 16 fund+ 2 asym: n D6 connecting two 1/2 NS5s on a pair of O8+8D8 branes, LST
- vector+ 1 asym + 1 sym: n D6 connecting two 1/2 NS5s on $O8^-$ and $O8^+$, LST

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- Bhardwaj: 1502.06559

Global Anomaly free condition

- Bershadsky and Vafa, 9703167
- Global Anomaly: SU(2),SU(3),G_2,

$$\pi_6(SU(2)) = \mathbf{Z}_{12},\ \pi_6(SU(3)) = \mathbf{Z}_6,\ \pi_6(G_2) = \mathbf{Z}_3$$

$$n_2 - 4 = 0 \text{ mod } 2 \text{ for } SU(2)$$
$$n_3 - n_6 = 0 \text{ mod } 6 \text{ for } SU(3)$$
$$n_7 - 1 = 0 \text{ mod } 3 \text{ for } G_2$$

Simple gauge group without no quartic

- SU(2): n_2=4,10,16 (LST)
- SU(3): n_3=0,6,12,18 (LST)
- G_2 : n_7=1,4,7,10 (LST)
- SO(8): n(v,c,s) =0,1,2,3,4(LST)
- F_4: n_{26}=0,1,2,3,4,5(LST)
- E_6: n_{27}=0,1,2,3,4,5,6(LST)
- E_7: $n_{56/2}$=0,1,2,3,4,5,6,7,8(LST)
- E_8

N M5s on ADE singularities

N array of (G,G) conformal matters with G in insides get gauged

A_N = N D6 with n NS5 : [SU(N)]-SU(N)-SU(N)…-[SU(N)]

D_{N+4}= [SO(2N+8)]-Sp(N)-SO(2N+8)-Sp(N)….Sp(N)-[SO(2N+8)]

E_6: +.. (N-1) E_6 gauge theories +2 E_6 global symmetries:

[E_6] -T-SU(3)-T-E_6-T -SU(3)-… … -SU(3)-T- [E_6]

E_7: (N-1) E7 gauge theories+ 2E_7 global symmetries..,

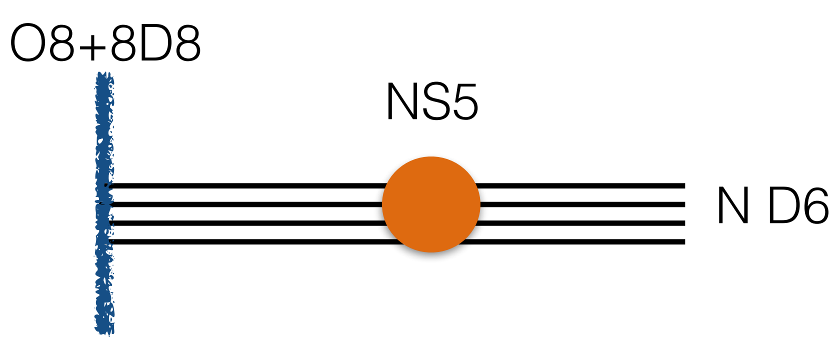
[E7]-T-SU(2)-SO(7)-SU(2)-T- E7-T-……… -T-[E7]

E_8: (N-1) E_8 gauge theories +2E_8 global symmetries

[E_8]-T-T-SU(2)-G_2-T-F_4-T-G_2-SU(2)-T-T-E8-T-……-T-T-[E8]

From 6d SCFTs to 5d SCFTs

- Sp(N)-[2N+8] + tensor
- [8]-Sp(N)—[2N]: or Sp[N]-[SO(4N+16)]
  - N=0: T-[8] : E_8-string theory
  - N=2: Sp(1)-[10] with SO(20) symmetry



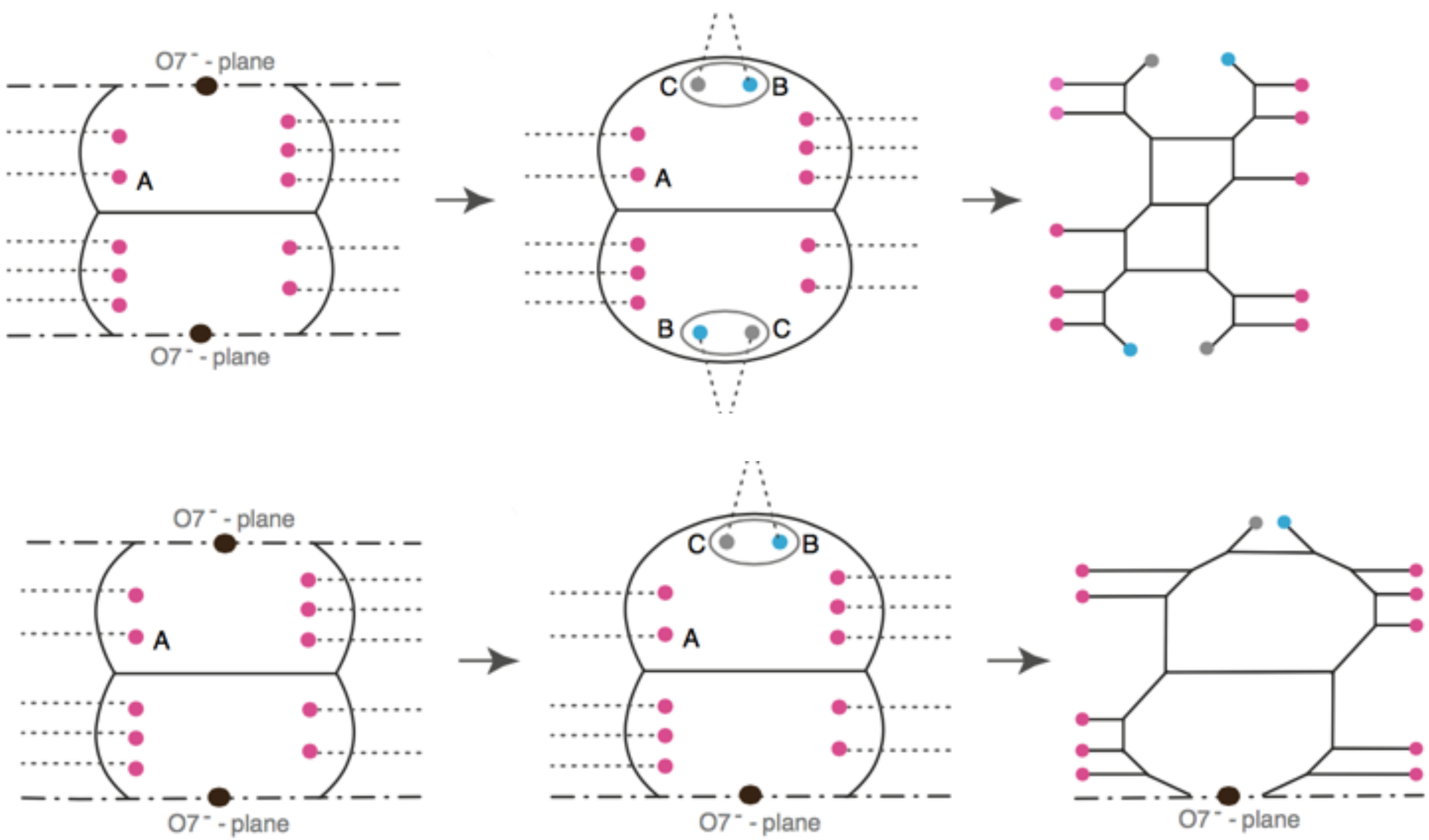
Circle compactification to 5d

H.Hayashi,S.Kim,K.L.,F.Yagi 1509.03300

6d Sp(1)+ tensor + 10 hypermultiplets
5d SU(3) with N_f=10
5d Sp(2) with N_f= 10

5d SO(20) x $U(1)_{KK}$ global symmetry

$1/R_{KK}$ tr F^2



5d SCFT

- Nonrenormalizable, $[g_{YM}^2]$= Length=1/M
- Infinite coupling limit: conformal fixed point (well-defined)
- YM term= MF^2 = relevant deformation
- BPS instanton becomes massless at the conformal fixed point.
- There exists one U(1) global charge for instantons of each gauge group.
- The global symmetry is a product of flavor symmetry G_f x U(1) can be enhanced in the strong coupling limit. (Seiberg 96)

SU(2), n_2=0,1,2,,,,7

- theta term: θ=0,π (Seiberg,Intriligator, Morrison….) for n_2=0
- Seiberg: the enhancement of global symmetry $SO(2n_2) \rightarrow E_{(n_2+1)}$
- E_0,$\tilde{E}_1$,E_1=SU(2),E_2=SU(2)xU(1),E_3=SU(3)xSU(2), E_4=SU(5),E_5=SO(10), E_6,E_7,E_8
- A (p,q) 5 brane terminates at [p,q] 7 brane (Dewolfe, Hanany,Iqbal,Katz)
- superconformal index: localization, instanton operators
  - Hee-Cheol Kim,Sung-Soo Kim, KL(1206.6781), C Hwang, J. Kim, Seok Kim, J. Park 1406.6793
- topological vertex & branes
  - Bao, Mitev, Pomoni, Taki, Yagi 1310.3841, H. Hayashi, H.C. Kim, Nishinaka 1310.3854, S.S. Kim, M. Taki, F. Yagi 1504.03672
- n_2=8 case: 6d tensor theory + 8 hypermultiplet: E_8 x $U(1)_{\text{Kaluza-Klein}}$
  - M5 brane exploring the E_8 wall

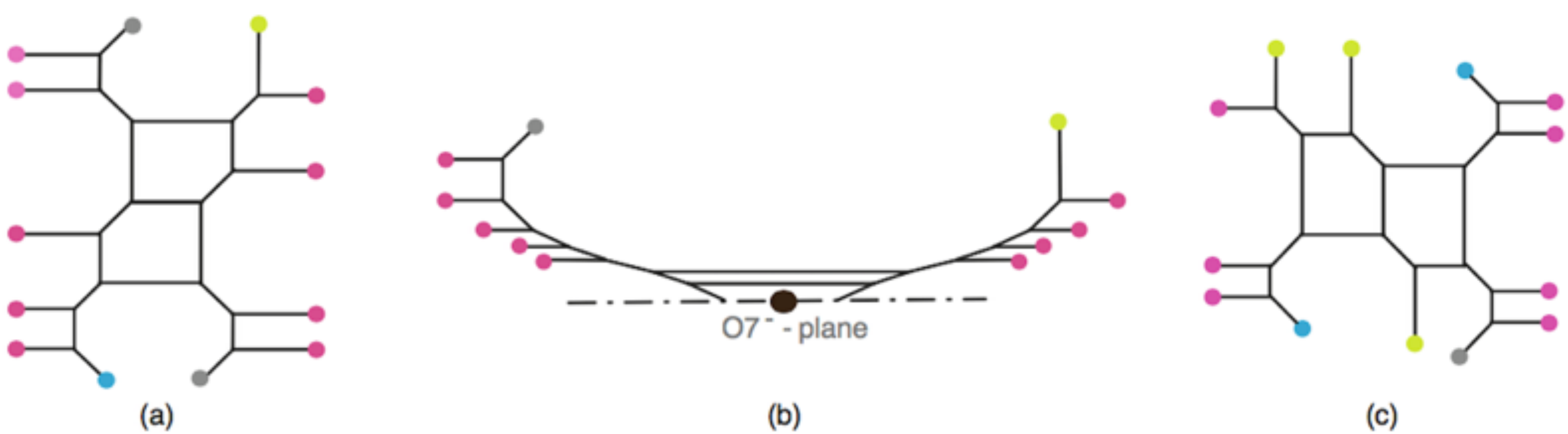
5d SU(N)

- SU(N)+ N_f fundamental hyper + κ-Chern-Simons level (tr AFF+..)
- N_f + 2|κ| =even integer: otherwise, anomalous
- N+2|κ| ≦ 2N (Intriligator, Morrison, Seiberg)
- N+2|κ| ≦ 2N+4, Enhanced global symmetry at UV (Bergman, Zafrir)
    - (Sung-Soo Kim, Masato Taki, Futoshi Yagi, Tao-diagram..)
    - (H. Hayashi, S.S. Kim, KL, M. Taki,F. Yagi 1505.04439)
    - (K.Yonekura, Gaiotto and H. Kim, 2015)
- $SU(N)_0$, N_f =2N+4 with finite coupling: 6d completion

mass-decoupling limit and dualities

5d $SU(3)_{1/2}$+ N_f=9 , Sp(2)+ N_f=9 and κ=0,   [3]-SU(2)-SU(2)-[4]

SO(20) global symmetry



5d SU(3)κ Enhanced Global Symmetry

For total rank of G = N_f +1, abelian part may be added
N+2|κ| ≦ 10

N_f	$G_{\|\kappa\|}$ (κ is the Chern-Simons level)
10	$SO(20)_0$
9	$SO(20)_{\frac{1}{2}}$
8	$SU(10)_0$, $[SO(16) \times SU(2)]_1$
7	$[SU(8) \times SU(2)]_{\frac{1}{2}}$, $SO(14)_{\frac{3}{2}}$
6	$[SU(6) \times SU(2) \times SU(2)]_0$, $SU(7)_1$, $SO(12)_2$
5	$[SU(5) \times SU(2)]_{\frac{1}{2}}$, $SU(6)_{\frac{3}{2}}$, $SO(10)_{\frac{5}{2}}$
4	$SU(4)_0$, $[SU(4) \times SU(2)]_1$, $SU(5)_2$, $SO(8)_3$
3	$SU(3)_{\frac{1}{2}}$, $[SU(3) \times SU(2)]_{\frac{3}{2}}$, $SU(4)_{\frac{5}{2}}$, $SO(6)_{\frac{7}{2}}$
2	$SU(2)_0$, $SU(2)_1$, $[SU(2) \times SU(2)]_2$, $SU(3)_3$, $SO(4)_4$
1	$SU(2)_{\frac{5}{2}}$, $SU(2)_{\frac{7}{2}}$
0	$SU(2)_3$

5d SU(N)κ Enhanced Global Symmetry

For total rank of G = N_f +1, abelian part may be added
N+2|κ| ≦ 2N+4

N_f	$G_{\|\kappa\|}$
$2n+4$	$SO(4n+8)_0$
$2n+3$	$SO(4n+8)_{\frac{1}{2}}$
$2n+2$	$SU(2n+4)_0$, $\left[SO(4n+4)\times SU(2)\right]_1$
$2n+1$	$\left[SU(2n+4)\times SU(2)\right]_{\frac{1}{2}}$, $SO(4n+2)_{\frac{3}{2}}$
$2n$	$\left[SU(2n)\times SU(2)\times SU(2)\right]_0$, $SU(2n+1)_1$, $SO(4n)_2$

Conclusion

- A lot more to be learned about 5d and 6d SCFTs and LST
- Further relations between 5d and 6d SCFTs
- 6d (2,0) and (1,0) little string theories
- Lessons on 1,2,3,4 dim quantum field theories