Theory Prospective on Neutrino Masses, Mixing, Oscillations and Leptonic CP Violation

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Hsinchu, Taiwan
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There have been remarkable discoveries in neutrino physics in the last ~ 17 years.

Compellings Evidence for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of $\mu-$like events**

Dominant $\nu_\mu \rightarrow \nu_\tau$ K2K, MINOS, T2K; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

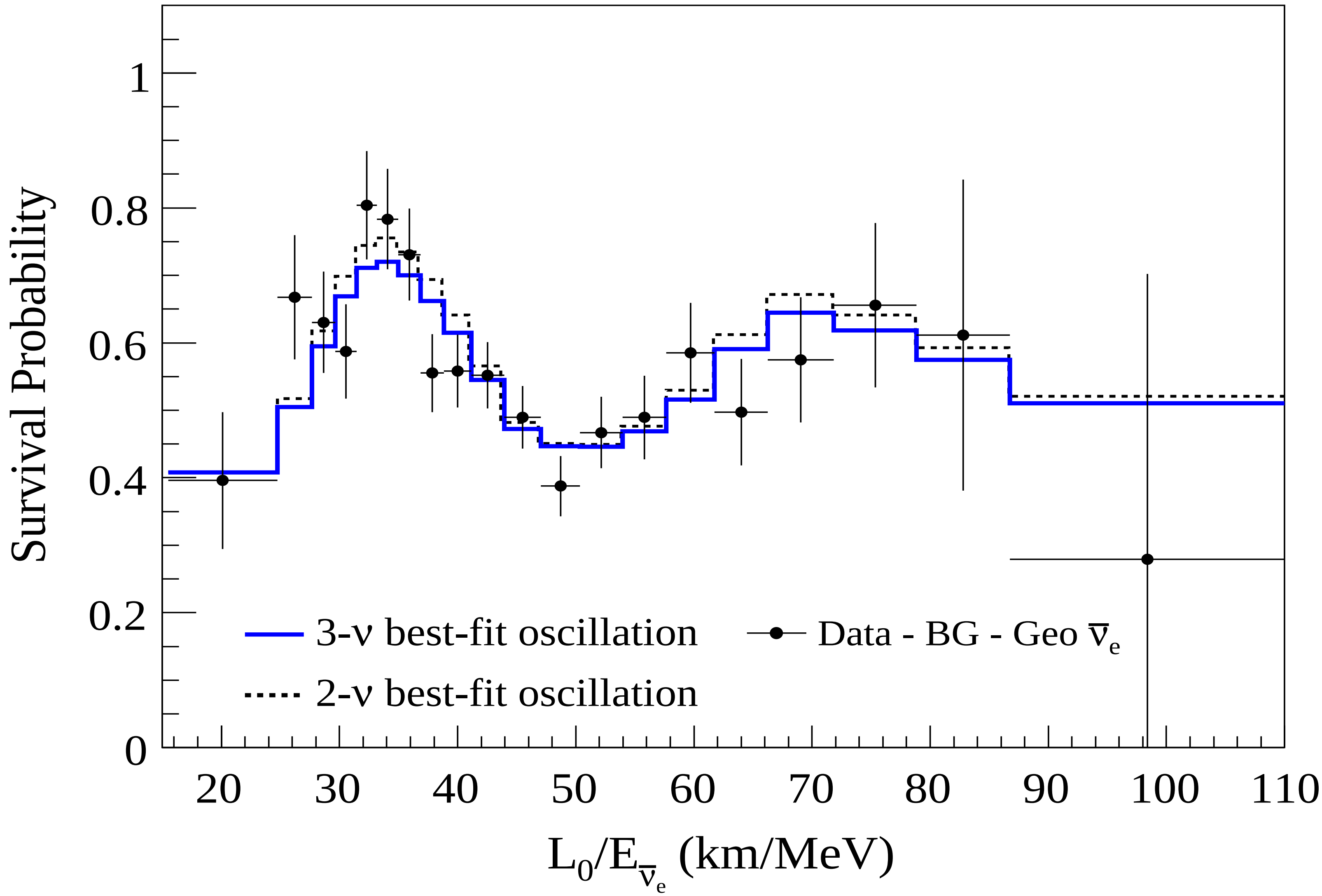
Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ **BOREXINO**

$-\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

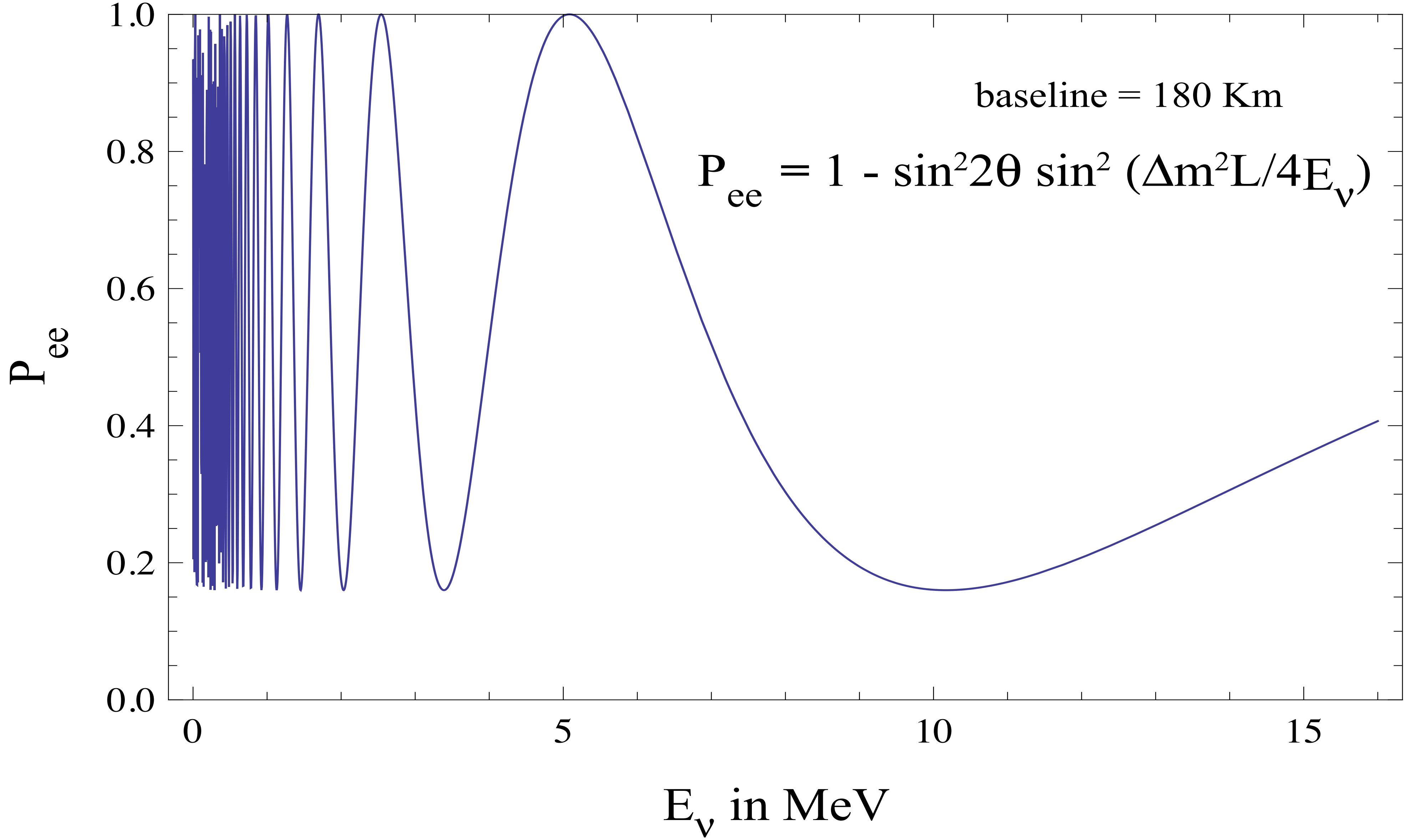
Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_μ from accelerators): $\nu_\mu \rightarrow \nu_e$



KamLAND: $L/E-$Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, E $=$ (1.8 - 10) MeV)

$\nu_e \to \nu_e$


baseline = 180 Km
P_ee = 1 - sin²2θ sin² (Δm²L/4E_ν)
1.0
0.8
0.6
0.4
0.2
0.0
P_ee
0
5
10
15
E_ν in MeV

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^{*}_{lj}\,|\nu_j>\,, \quad \nu_j :\; m_j \neq 0\,; \quad l = e,\mu,\tau\,; \quad n \geq 3\,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\,\nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x) :\; m_j \neq 0\,; \quad l = e,\mu,\tau\,.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

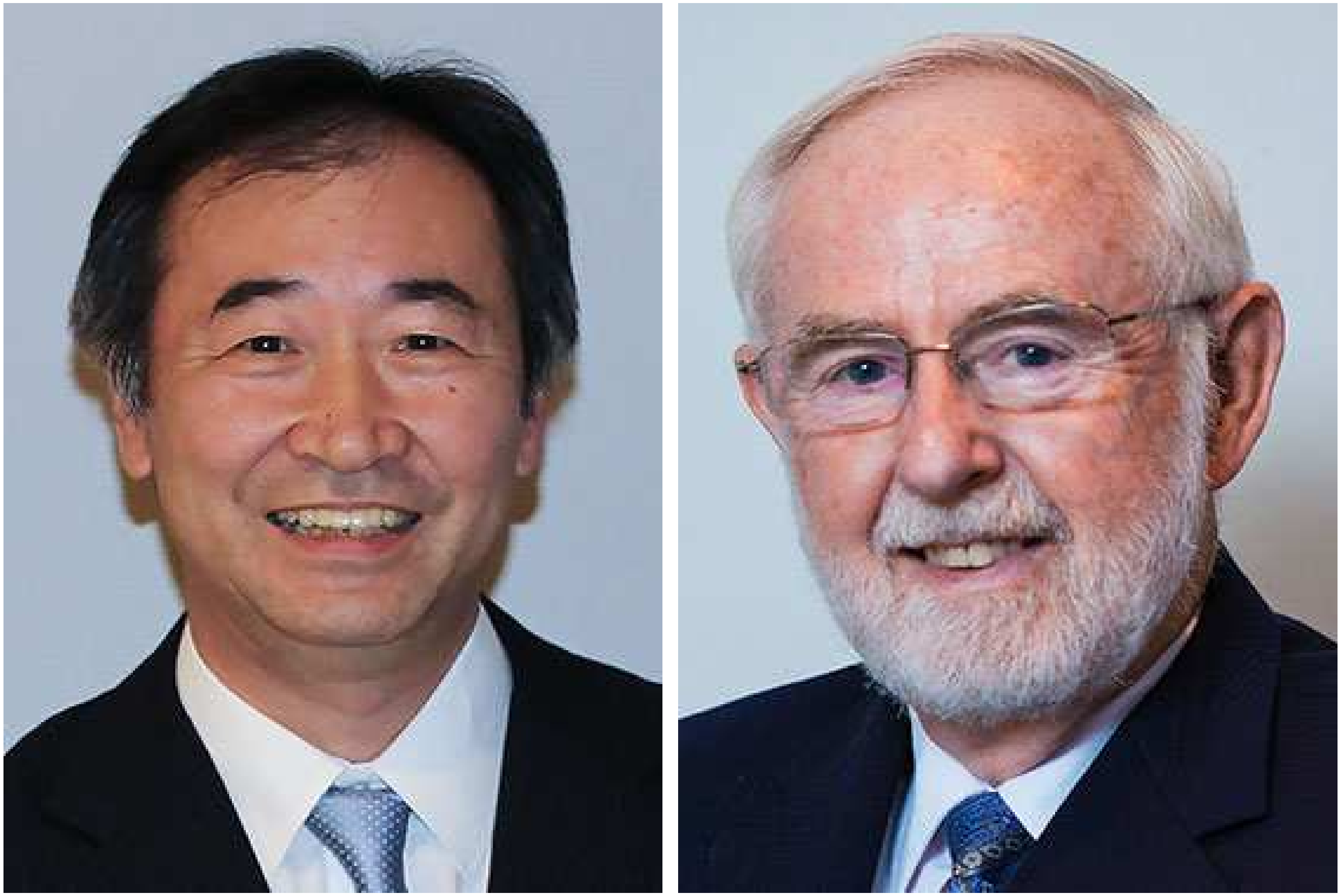
ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 1$ eV.

The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x)\gamma_\alpha \,(1-\gamma_5)\, \nu_{l\mathrm{L}}(x)\, W^\alpha(x) + \text{h.c.}\,,$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\; m_j \neq 0\,; \quad l = e, \mu, \tau\,.$$



Dr. T. Kajita, Prof. A. McDonald, Nobel Prize for Physics winners, 2015

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Kam-Biu Luk and the Daya Bay Collaboration


Yifang Wang and the Daya Bay Collaboration


Koichiro Nishikawa and the K2K and T2K Collaboration


Atsuto Suzuki and the KamLAND Collaboration


Arthur B. McDonald and the SNO Collaboration


Takaaki Kajita and the Super K Collaboration


Yoichiro Suzuki and the Super K Collaboration

These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q,\ q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2}$

These discoveries suggest the existence of New Physics beyond that of the ST.

The New Physics can manifest itself (can have a variety of different "flavours"):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...).

- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.

- In the Majorana nature of massive neutrinos.

- In the existence of LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.

- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos N_j, doubly charged scalars,...

- In the existence of new (FChNC, FCFNSNC) neutrino interactions.

- In the existence of "unknown unknowns"...

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,...} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly"), data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"); tests (STEREO, BOREXINO,...) under way).

ii) $M_{4,5,...} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
$M_{4,5,...} \sim (10^9 - 10^{13})$ GeV, "classical" seesaw models.

All compelling data compatible with 3-ν mixing:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (al least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu$, $n = 4, 5, ...$).

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x),\quad |\xi_k|^2 = 1,\quad C^{-1}\gamma_\mu C = -\ \gamma_\mu^T$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\text{el}} = 0,\ L_l = 0,\ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$−Dirac, $\chi(x)$−Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0\ ,\quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0\ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta}\ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi\ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP}\ \chi(x)\ U^{-1}_{CP} = \eta_{CP}\ \gamma_0\ \chi(x'),\quad \eta_{CP} = \pm i\ .$$

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.47\ (2.42) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437\ (0.455)$, NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234\ (0.0240)$, Capozzi et al. NO (IO).

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

• Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2\theta_{13} = 0.0241$ (0.0244), NO (IO).

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.47$ $(2.42) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0240), NH (IH).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = 2.6\%$, $1\sigma(\sin^2\theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2\theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m^2_{21})$: $(6.99 - 8.18) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.259 - 0.359)$;
- $3\sigma(|\Delta m^2_{31(23)}|)$: $2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV^2;
$3\sigma(\sin^2\theta_{23})$: $0.374(0.380) - 0.628(0.641)$;
- $3\sigma(\sin^2\theta_{13})$: $0.0176(0.0178) - 0.0295(0.0298)$.

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

• $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m^2_{31}(NO) = - \Delta m^2_{32}(IO)$$

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

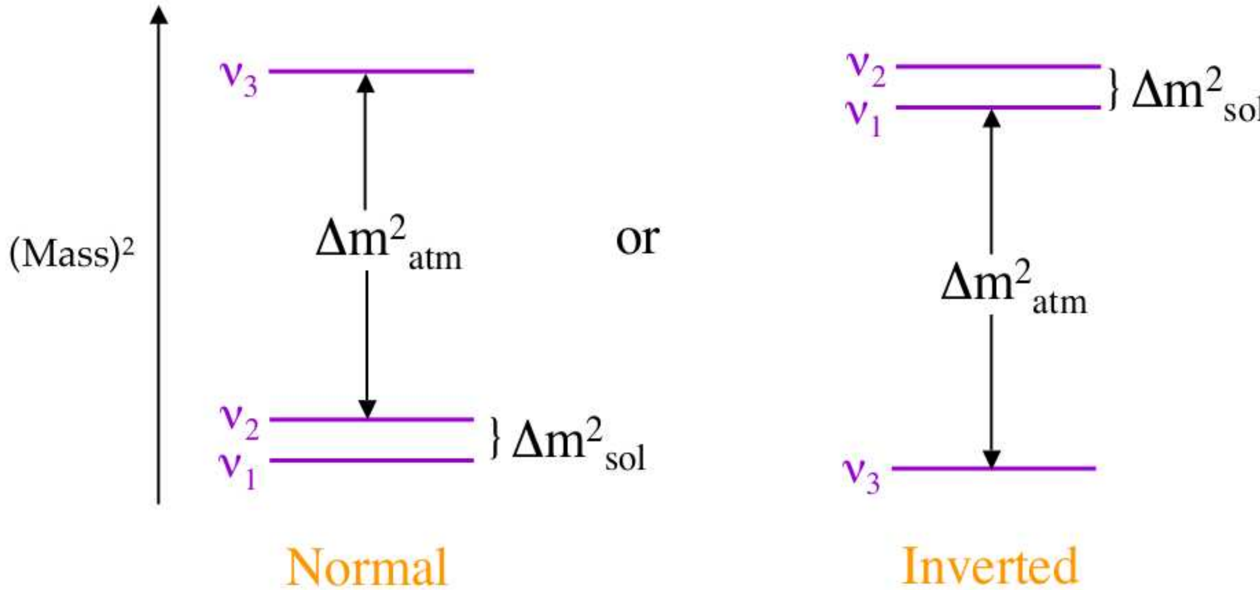
$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \; m^2_{1,2,3} >> \Delta m^2_{\text{atm}}, \; \text{QD}; \; m_j \gtrsim 0.10 \text{ eV}.$$

• $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;

• $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}$, $m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;

The (Mass)^2 Spectrum

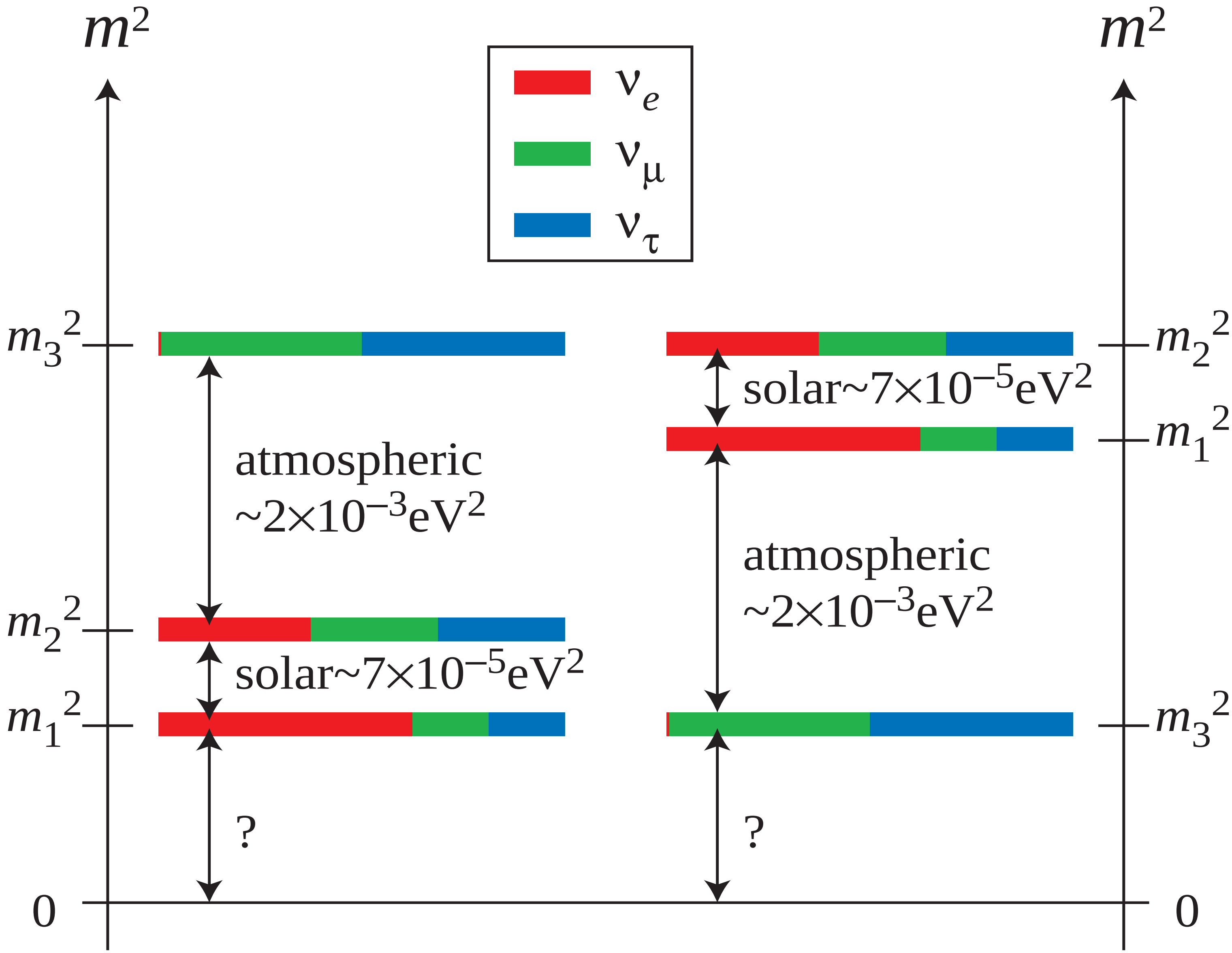


$\Delta m^2_{sol} \cong 7.6 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.4 \times 10^{-3}\ eV^2$

Are there *more* mass eigenstates, as LSND suggests, and MiniBooNE recently hints?

3

Due to B. Kayser



S. King, Ch. Luhn, 2013

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

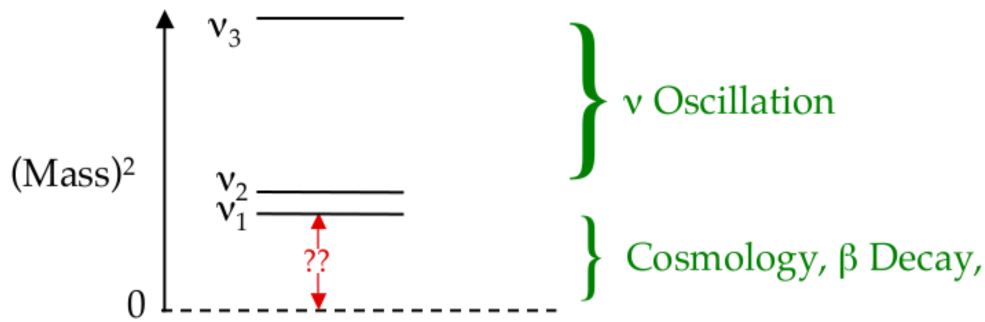
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

$$\text{Oscillation Data} \Rightarrow \sqrt{\Delta m^2_{atm}} < \text{Mass[Heaviest } \nu_i\text{]}$$

4

Due to B. Kayser

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on $^3\text{H} \to ^3\text{He} + e^- + \bar{\nu}_e$:

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.})$$

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming KATRIN experiment is planned to reach sensitivity

$$\text{KATRIN:} \quad m_{\nu_e} \sim 0.2 \text{ eV}$$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
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Manchester
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Birmingham
Cardiff
London
Celtic Sea
Irish Sea
Guernsey (U.K.)
Jersey (U.K.)
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GREECE
Aegean Sea
İzmir
BALEARIC ISLANDS
Cagliari
Athens
Sevilla
Ionian Sea
Palermo
Mediterranean Sea
Gibraltar
Málaga
Sicily
Rhodes
Ceuta (SPAIN)
Alborán Sea
Algiers
Crete
Melilla (SPAIN)
Oran
Tunis
Scale 1: 19,500,000
Rabat
Valletta
MALTA
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
Casablanca
ALGERIA
TUNISIA
300 Kilometers
300 Miles
MOROCCO

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + ΛCDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

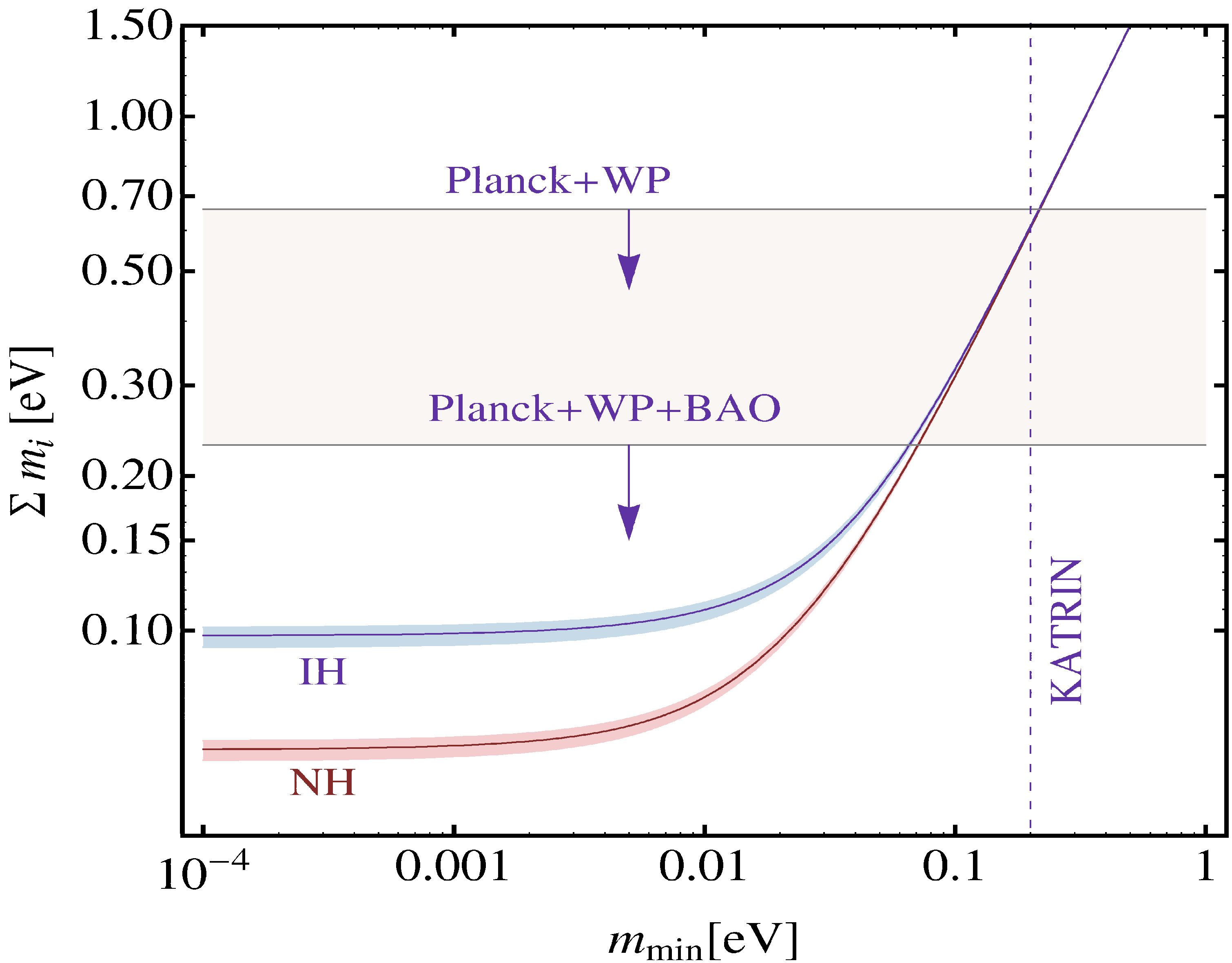
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \qquad \delta \cong (0.01 - 0.04) \text{ eV}.$$

NH: $\sum_j m_j \leq 0.05$ eV (3σ);

IH: $\sum_j m_j \geq 0.10$ eV (3σ).

Mass and Hierarchy from Cosmology



Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of $\text{sgn}(\Delta m^2_{\text{atm}})$, type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or $\min(m_j)$.

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, θ_{12}, Δm^2_{atm}, θ_{23}, θ_{13}

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in U_{PMNS} ;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos ($(\beta\beta)_{0\nu}$-decay exps: GERDA, CUORE, EXO, KamLAND-Zen, SNO+, SuperNEMO, MAJORANA, AMORE,...).

- determination of the neutrino mass ordering;

- determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology);

- determination of the status of the CP symmetry in the lepton sector (T2K, NOνA; DUNE, T2HK)

Determining the ν−Mass Ordering (sgn($\Delta m^2_{\rm atm}$))

- **Reactor $\bar{\nu}_e$ Oscillations in vacuum (JUNO, RENO50).**
- **Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects) (ORCA, PINGU (IceCube), HK, INO).**
- **LBL ν−oscillation experiments (T2K + NOνA; DUNE); designed to search also for CP violation.**
- ^{3}H β**-decay Experiments (sensitivity to** 5×10^{-2} **eV) (NH vs IH).**
- $(\beta\beta)_{0\nu}$−**Decay Experiments; ν_j− Majorana particles (NH vs IH).**
- **Cosmology: $\sum_j m_j$ (NH vs IH).**
- **Atomic Physics Experiments: RENP.**


Guangzhou
Guang Zhou
Dongguan
Huizhou
Zhaoqing
CNS
2.5 h drive
Shen Zhen
Huizhou NPP
Lufeng NPP
Daya Bay
NPP
Zhongshan
Zhu Hai
Zhujiang River Estuary
Hong Kong
Hong Kong
Macau
Macau
JUNO site
53 km
53 km
Taishan NPP
Yangjiang NPP

Large $\sin\theta_{13} \cong 0.16$ (Double Chooz, Daya Bay, RENO) - far-reaching implications for the program of reserach in neutrino physics:

- For the determination of the type of $\nu-$ mass spectrum (or of sgn($\Delta m^2_{\rm atm}$)) in neutrino oscillation experiments.

- For understanding the patern of the neutrino mixing and its origins (symmetry, etc.?).

- For the predictions for the $(\beta\beta)_{0\nu}$-decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).

Large $\sin\theta_{13} \cong 0.16$ + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;

- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU is

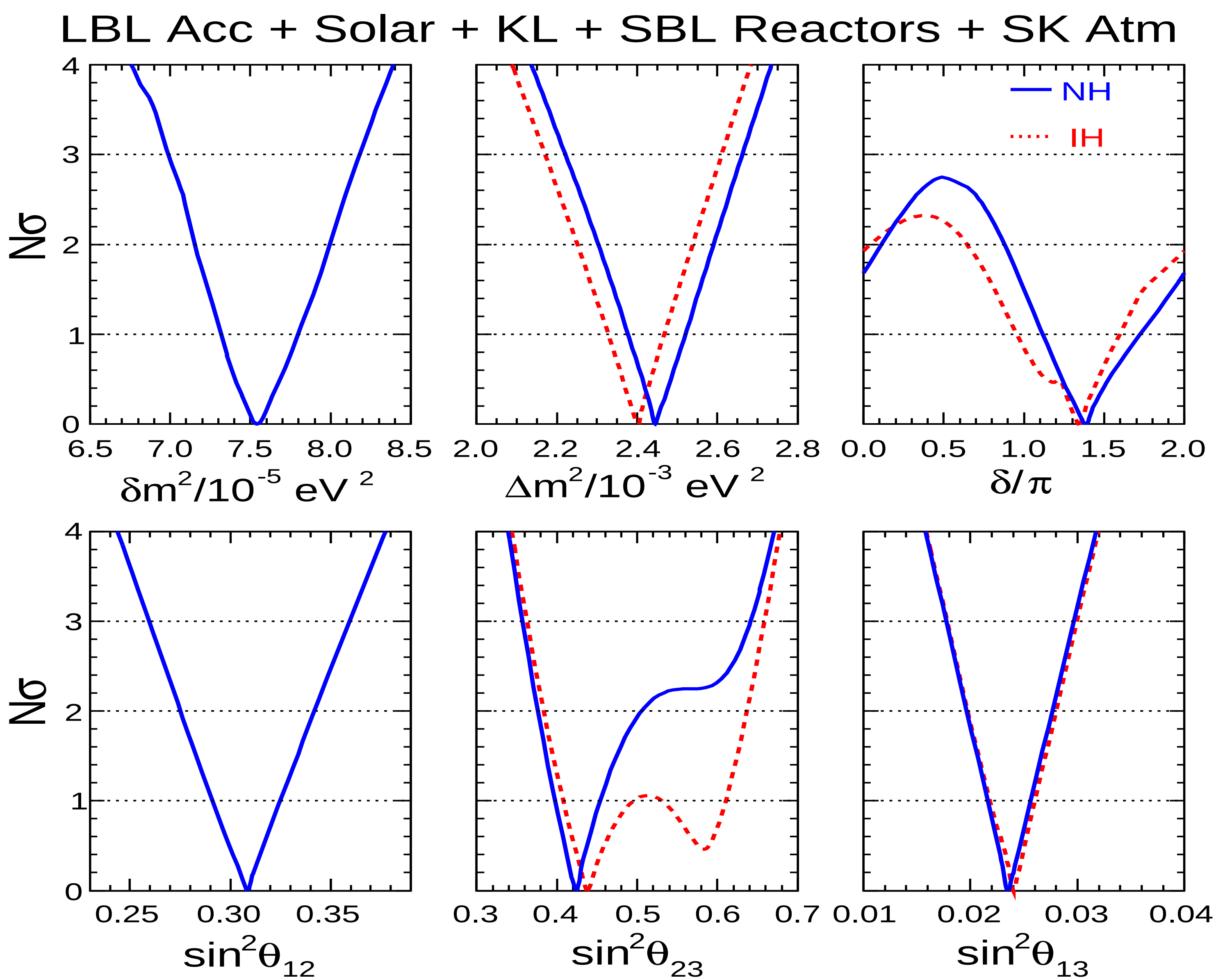
$$|\sin\theta_{13}\,\sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

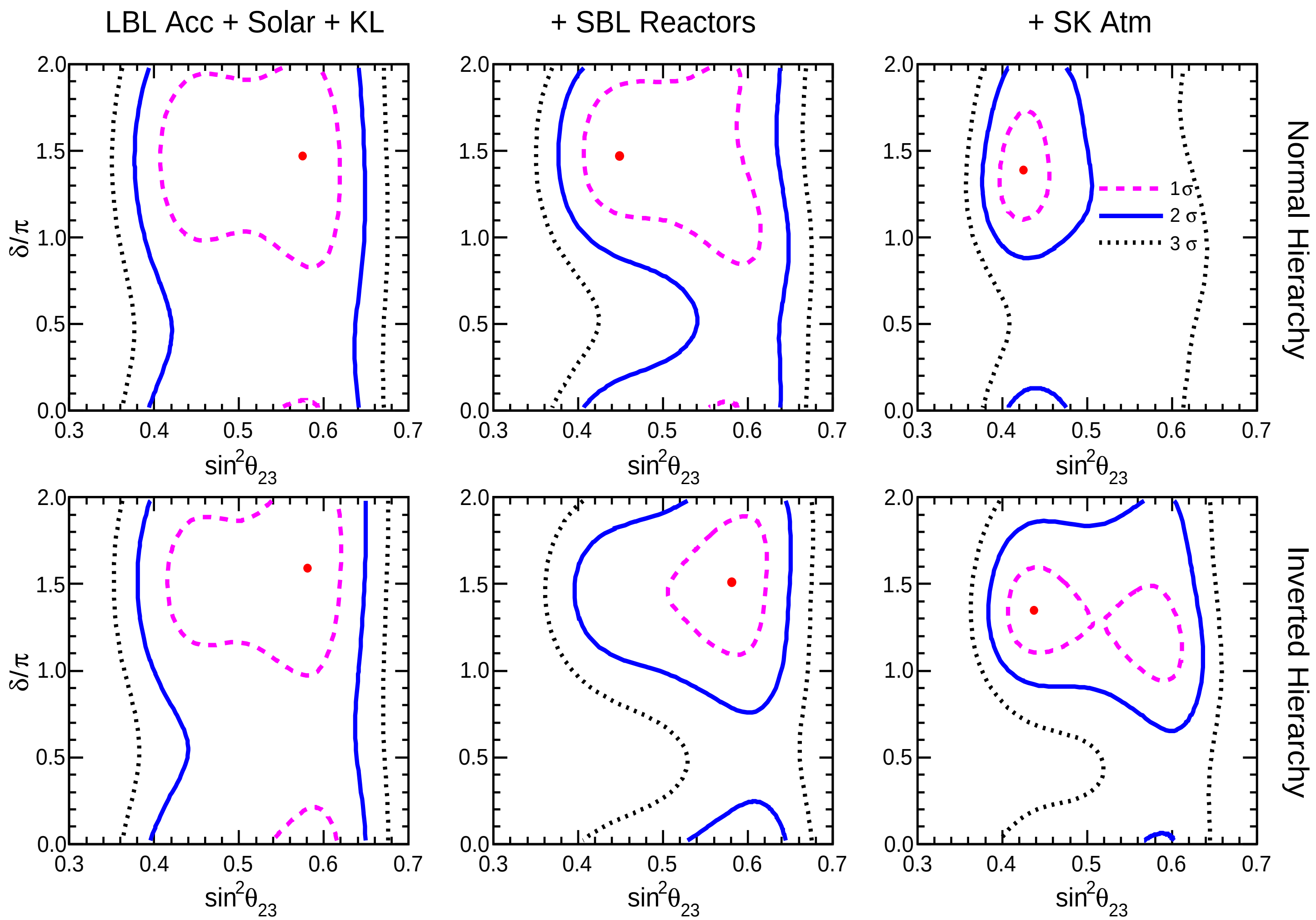
$$\delta \cong 3\pi/2?$$

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\}$$

$$= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

• November 11, 2011, Double Chooz: 2σ evidence for $\theta_{13} \neq 0$.

• March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.

• April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.

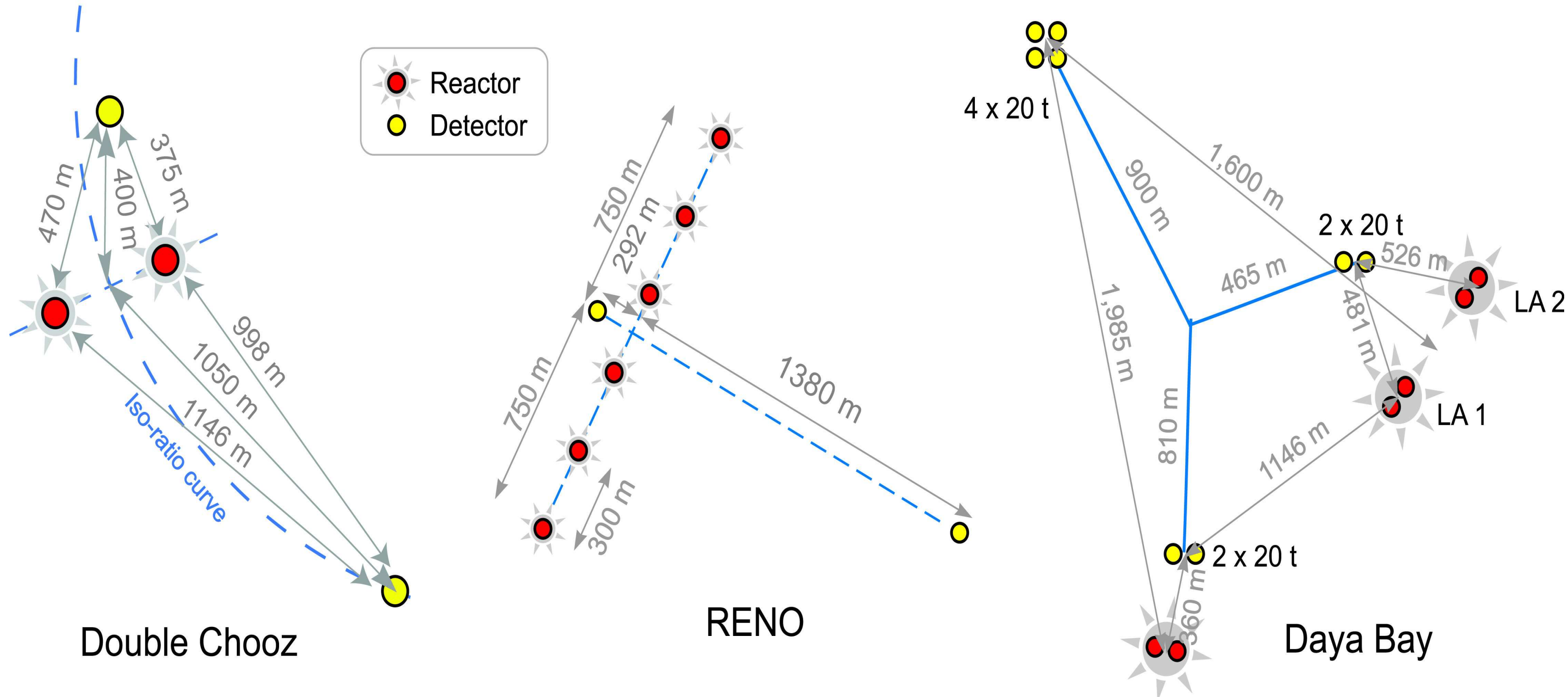
• Daya Bay, May 2015:
$\sin^2 2\theta_{13} = 0.084 \pm 0.005$.

• RENO, April 2015 (WIN 2015):
$\sin^2 2\theta_{13} = 0.087 \pm 0.008\ (stat.) \pm 0.008$.

• Double Chooz, 2014:
$\sin^2 2\theta_{13} = 0.090^{+0.032}_{-0.029}\ (0.092^{+0.033}_{-0.029})$.

$$P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m^2_{31(32)}; \theta_{12}, \Delta m^2_{21}) \cong$$

$$1 - \sin^2 2\theta_{13} \sin^2(\frac{\Delta m^2_{31(32)}}{4E} L),$$ no dependence on θ_{23}, δ.



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Kawasaki
Nagoya
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

T2K: Search for $\nu_{\mu} \rightarrow \nu_{e}$ oscillations

T2K: first results March 2011 (2 events);
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ;
July, 2013 (28 events).

For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (1.7), best fit.

This value is by a factor of ~ 1.6 (1.9) bigger than the value obtained in the Daya Bay and RENO experiments.

$$P^{3\nu}_{m}(\nu_{\mu} \rightarrow \nu_{e}) = P^{3\nu}_{m}(\theta_{13}, \Delta m^2_{31(32)}, \theta_{12}, \Delta m^2_{21}, \theta_{23}, \delta).$$

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E}\,,\;\; A = \sqrt{2}G_{\mathrm{F}}N_e^{man}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \to \bar{\nu}_e$: $\delta,\;\; A \to (-\delta),\;\; (-A)$

The first results on $\nu_\mu \rightarrow \nu_e$ oscillations from NOνA (August 6, 2015; 6 (12) events) are compatible with, and somewhat strengthened, the hint that $\delta \cong 3\pi/2$ (A. Marone, talk at TAUP 2015).

The Quest for Nature's Message

With the observed pattern of neutrino mixing Nature is sending us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. The message can have two completely different contents: it can read

ANARCHY or SYMMETRY.

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

Understanding the Pattern of Neutrino Mixing.

Predictions for the CPV Phase δ.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\rm PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20, \quad \theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 \mp 0.10.$
- $U_{\rm PMNS}$ due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi,\omega) U_{\text{TBM,BM,LC,...}}\; \bar{P}(\xi_1, \xi_2)\,,$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM,BM,LC,...}}\; \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi,\omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix}; \quad \mu - \tau \ \text{ symmetry}: \ \ \theta^{\nu}_{23} = \mp\pi/4\,;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta^{\nu}_{12} = \pi/6\,.$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2 + r)^{-1} \cong 0.276$, $r = (1 + \sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3 - r)/4 \cong 0.345$.

• $U_{\rm TBM}$: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s^2_{23} = 0.5$ must be corrected .

• $U_{\rm BM}$: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$, $s^2_{12} = 1/2$ and possibly $s^2_{23} = 1/2$ must be corrected.

$U_{\rm TBM(BM)}$: Groups A_4, T' (S_4), ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

• $U_{\rm GRA}$: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

• $U_{\rm LC}$: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

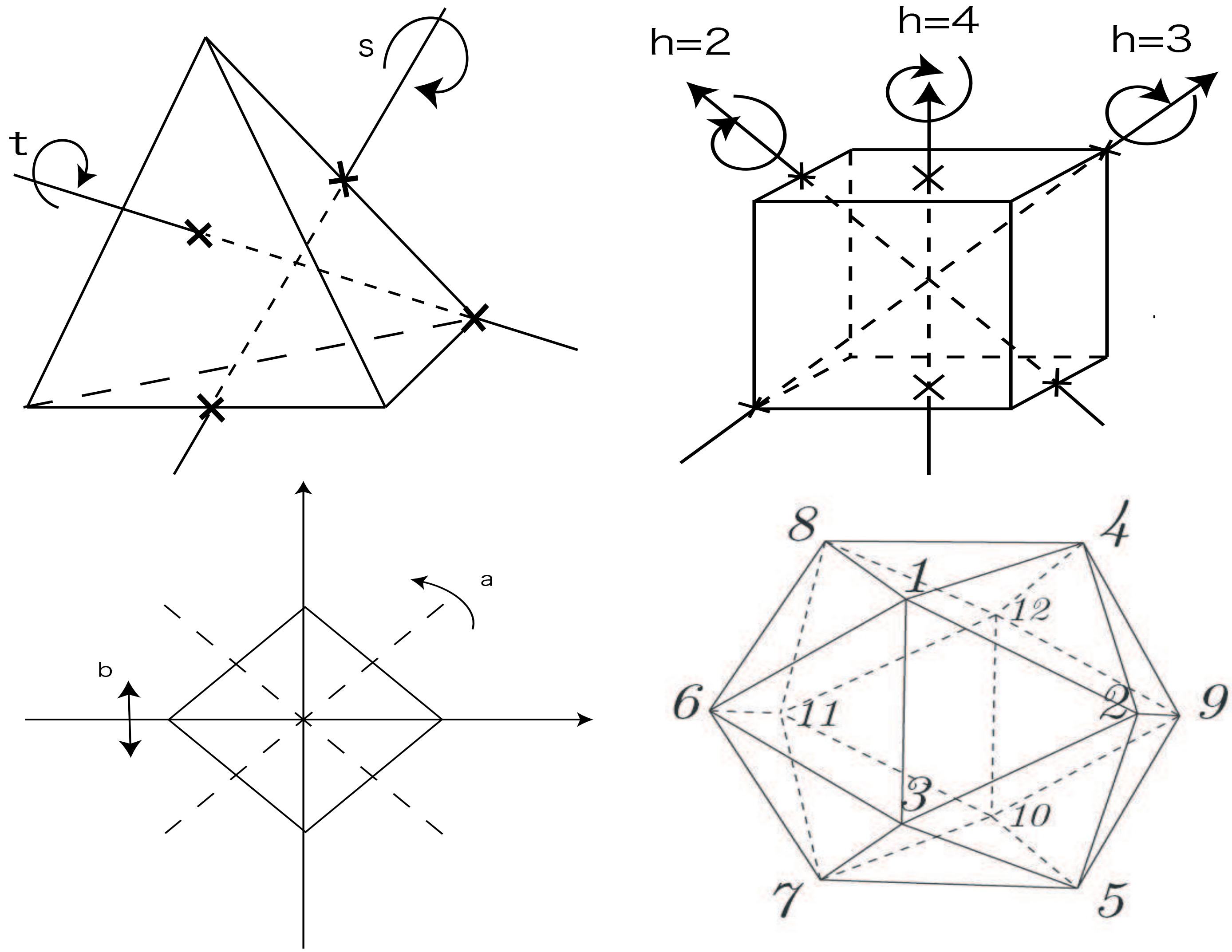
S.T.P., 1982

• $U_{\rm LC}$: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• $U_{\rm GRB}$: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

• $U_{\rm HG}$: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta^{\nu}_{13} = 0$, $\theta^{\nu}_{23} = \mp\pi/4$.
They differ by the value of θ^{ν}_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^{\nu}_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.



Examples of symmetries: A_4, S_4, D_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	$S,\ T,\ U$	$\mathbf{1},\ \mathbf{1'},\ \mathbf{2},\ \mathbf{3},\ \mathbf{3'}$
A_4	12	$S,\ T$	$\mathbf{1},\ \mathbf{1'},\ \mathbf{1''},\ \mathbf{3}$
T'	24	$S,\ T\ (R)$	$\mathbf{1},\ \mathbf{1'},\ \mathbf{1''},\ \mathbf{2},\ \mathbf{2'},\ \mathbf{2''},\ \mathbf{3}$
A_5	60	$S,\ T$	$\mathbf{1},\ \mathbf{3},\ \mathbf{3'},\ \mathbf{4},\ \mathbf{5}$
D_{10}	20	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4$
D_{12}	24	$A,\ B$	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{2}_1,\ \mathbf{2}_2,\ \mathbf{2}_3,\ \mathbf{2}_4,\ \mathbf{2}_5$

Number of elements, generators and irreducible representations of some discrete groups.

None of the symmetries leading to $U_{\rm TBM}$, $U_{\rm BM}$ or other approximate forms of $U_{\rm PMNS}$ can be exact.

Which is the correct approximate symmetry, i.e., approximate form of $U_{\rm PMNS}$ (if any)?

In the cases of U_{ν} given by $U_{\rm TBM}$, $U_{\rm BM}$, etc. the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\rm TBM,BM}$, etc. and on the form of $U_{\rm lep}$, one obtaines diiffer- ent experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spec- trum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the ob- served patterns of neutrino mixing and Δm^2_{ij}.

What is the minimal U_{lep} providing the requisite corrections to $U_{\text{TBM,BM,LC,GRM,HGM}}$?

Predictions for δ

Assume:

- $U_{PMNS} = U^{\dagger}_{\text{lep}}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi, \omega) U_{\text{TBM,BM,GR,HG}} \bar{P}(\xi_1, \xi_2)$,

- $U^{\dagger}_{\text{lep}}$ - minimal, such that

i) $\sin\theta_{13} \cong 0.16$; BM: $\sin^2\theta_{12} \cong 0.31$;
ii) $\sin^2\theta_{23}$ can deviate significantly (by more than $\sin^2\theta_{13}$) from 0.5 (b.f.v. = 0.40-0.45) .

The "minimal" = simplest case ($SU(5) \times T'$,...)

$U_{\text{lep}} \cong O^{\ell}_{12}(\theta^{\ell}_{12})$; now $Q = \text{diag}(e^{i\varphi}, 1, 1)$;

$\sin^{\ell}\theta_{13}$, $\sin^{\ell}\theta_{23}$ - negligibly small ($SU(5) \times T'$,...).

Thus, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta^{\ell}_{12}, \varphi$ and θ^{ν}_{12}.

$\theta_{23} = \theta_{23}(\theta_{13})$, $\quad \delta = \delta(\theta_{12}, \theta_{13}, \theta^{\nu}_{12})$(!)

$U_{\text{BM},...}$: $\sin^2\theta_{12} \cong \sin^2\theta^{\nu}_{12} + \sin 2\theta^{\nu}_{12} \sin\theta_{13}\cos\delta + ...$,

The exact sum rule will be given later. The preceding approximate sum rule leads to a rather precise prediction for $\cos\delta$ only in the BM (LC) case.

U_{BM}: $\sin 2\theta^{\nu}_{12} = 1$ and $\sin^2\theta_{12} = (0.31 - 0.33)$ require $\cos\delta \cong -1$.

$$\sin^2\theta_{23} = \frac{1-2\sin^2\theta_{13}}{2(1-\sin^2\theta_{13})} \cong 0.5(1 - \sin^2\theta_{13}).$$

Problem for $U_{\text{TBM,BM,GRA(B),HG}}$ if $\sin^2\theta_{23} \cong 0.44-0.45$:

Larger correction to $\sin^2\theta^{\nu}_{23} = 0.5$ might be needed.

Next-to-Minimal case: $U_{\text{lep}} \cong U_{\text{lep}}(\theta^{\ell}_{12}, \theta^{\ell}_{23})$,
$Q = \text{diag}(1, e^{-i\psi}, e^{-i\omega})$.

"Standard" Ordering:
$U_{\text{lep}}(\theta^{\ell}_{12}, \theta^{\ell}_{23}) = O^T_{23}(\theta^{\ell}_{23}) O^T_{12}(\theta^{\ell}_{12})$ (GUTs typically);
in many theories - a consequence of $m_e^2 \ll m_\mu^2 \ll m_\tau^2$.

Standard Ordering

$$U = O_{12}(\theta^{\ell}_{12}) O_{23}(\theta^{\ell}_{23}) \text{diag}(1, e^{-i\psi}, e^{-i\omega}) O_{23}(\theta^{\nu}_{23}) O_{12}(\theta^{\nu}_{12}) \bar{P},$$

$$\bar{P} = \text{diag}(1, e^{i\xi_1}, e^{i\xi_2}).$$

Can be shown to be equivalent to:

$U = O_{12}(\theta^{\ell}_{12}) \text{diag}(1, e^{i\phi}, 1) O_{23}(\hat{\theta}_{23}) O_{12}(\theta^{\nu}_{12}) \bar{P}(\xi_1, \xi_2 + \beta)$

$\hat{\theta}_{23} = \hat{\theta}_{23}(\theta^{\ell}_{23}, \psi - \omega, \theta^{\nu}_{23}), \quad \phi = \phi(\theta^{\ell}_{23}, \psi, \omega, \theta^{\nu}_{23}).$

$\theta^{\nu}_{12} = \pi/4$ (BIM,LC), or $\sin^{-1}(1/\sqrt{3})$ (TBM), etc.

Thus, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta^{\ell}_{12}, \phi, \hat{\theta}_{23}$.

ϕ serves as a “source” for δ.

Expect $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$(!)

For arbitrary fixed θ^{ν}_{12} and any θ_{23}
("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right].$$

S.T.P., arXiv:1405.6006

This results is exact. For TBM and BM cases derived first in:

D. Marzocca, S.T.P., A. Romanino, M.C. Sevillia, arXiv:1302.0423

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.

In a complete self-consistent theory corrections are possible: $\theta^{e}_{13} \neq 0$ (see further), RG running effects (negligible for $m_0 \lesssim 0.01$ eV), etc.

Comparing the imaginary and real parts of $U^*_{e1}U^*_{\mu 3}U_{e3}U_{\mu 1}$ in the two parametrisations:

$$\sin\delta = -\frac{\sin 2\theta^{\nu}_{12}}{\sin 2\theta_{12}}\sin\phi\,,$$

$$\cos\delta = \frac{\sin 2\theta^{\nu}_{12}}{\sin 2\theta_{12}}\cos\phi\left(\frac{2\sin^2\theta_{23}}{\sin^2\theta_{23}\cos^2\theta_{13}+\sin^2\theta_{13}} - 1\right)$$

$$+\frac{\cos 2\theta^{\nu}_{12}}{\sin 2\theta_{12}}\frac{\sin 2\theta_{23}\sin\theta_{13}}{\sin^2\theta_{23}\cos^2\theta_{13}+\sin^2\theta_{13}}\,.$$

S.T.P., arXiv:1405.6006

The relations are exact.

In all cases BM, LC, TBM, GRA, GRB, HGM:

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij}:
$\delta \cong 263.5^{\circ}$ or 96.5°, $\cos\delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286.8^{\circ}$ or 73.2°;
$\cos\delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- GRBM case, b.f.v. of θ_{ij}: $\delta \cong 258.5^{\circ}$ or 101.5°;
$\cos\delta = -0.200$, $J_{CP} \mp 0.0333$.
- HGM case, b.f.v. of θ_{ij}: $\delta \cong 298.4^{\circ}$ or 61.6°;
$\cos\delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- BM, LC cases: $\delta \cong \pi$, $\cos\delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij}: F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos\delta$ or δ one can distiguish between different symmetry forms of $\tilde{U}_\nu$!

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, see, e.g., A. de Gouvea *et al.*, arXiv:1310.4340, P. Coloma *et al.*, arXiv:1203.5651.

Alternative "minimal" case
$U_{\rm lep} \cong O^{\ell}_{13}(\theta^{\ell}_{13})$; also in this case $Q = {\rm diag}(e^{i\varphi_{13}}, 1, 1)$;

Now, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta^{\ell}_{13}, \varphi_{13}$ and θ^{ν}_{12}.

Similarly we have: $\sin^2\theta_{23} = \frac{1}{2(1-\sin^2\theta_{13})}$.

Alternative Next-to-Minimal case: $U_{\rm lep} \cong U_{\rm lep}(\theta^{\ell}_{13}, \theta^{\ell}_{23})$,
$Q = {\rm diag}(1, e^{-i\tilde{\psi}}, e^{-i\tilde{\omega}})$.

"Standard" Ordering:
$U_{\rm lep}(\theta^{\ell}_{12}, \theta^{\ell}_{23}) = O^{T}_{23}(\theta^{\ell}_{23}) O^{T}_{13}(\theta^{\ell}_{13})$

For arbitrary fixed θ^{ν}_{12} and any θ_{23}
(alternative "minimal" and "next-to-minimal" cases):

$$\cos\delta = -\frac{\cot\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\Big[\cos 2\theta^{\nu}_{12}$$

$$+\left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \tan^2\theta_{23}\,\sin^2\theta_{13}\right)\Big]\,.$$

I. Girardi, S.T.P., A. Titov, arXiv:1504.00658

This results is exact. Differs only by the overall sign and by $\tan\theta_{23} \leftrightarrow \cot\theta_{23}$ from the sum rule for $\cos\delta$ discussed earlier.

Alternative "Minimal" case: $\sin^2\theta_{23} = \frac{1}{2(1-\sin^2\theta_{13})}$.

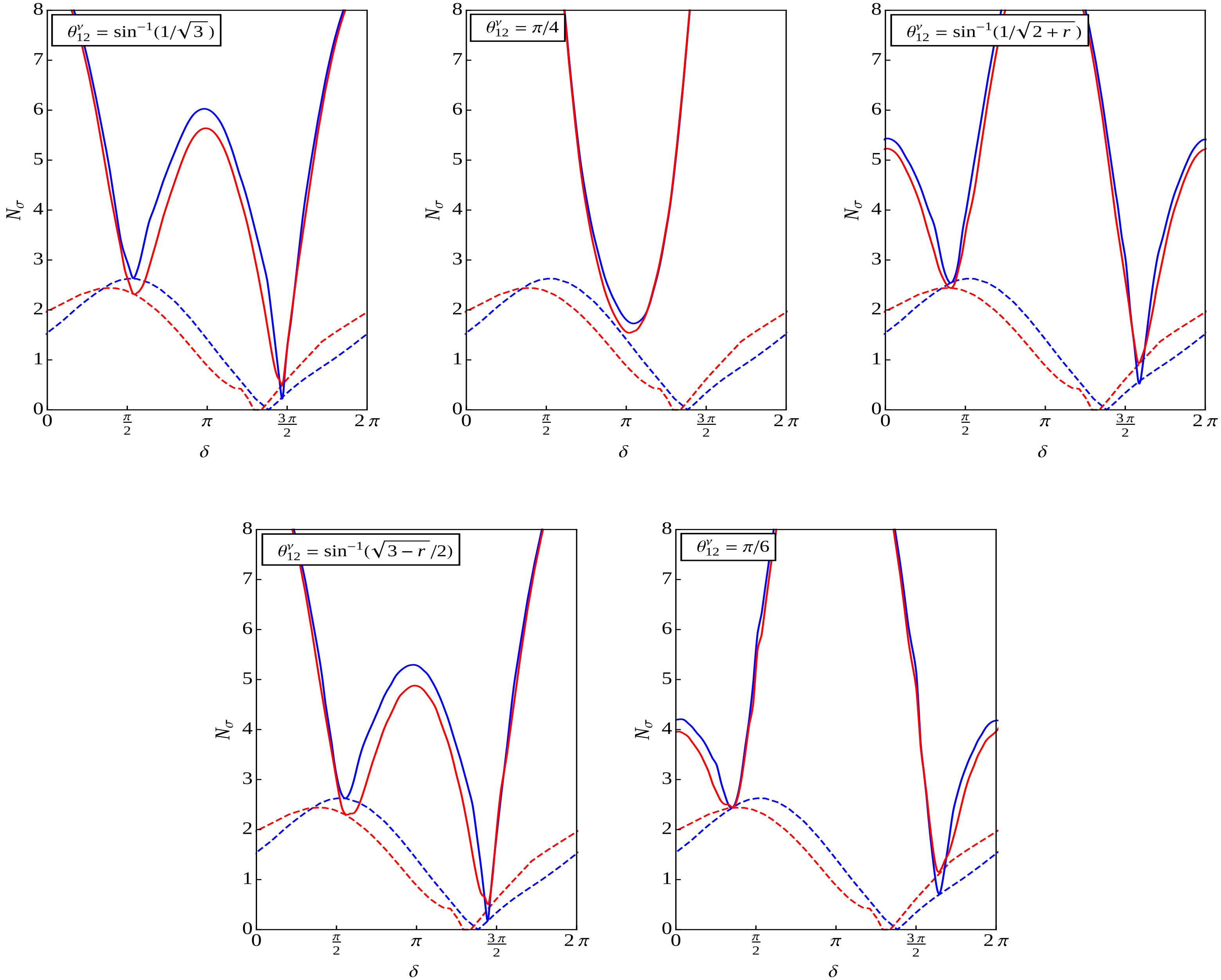
Statistical analysis, likelihood method;
input "data": $\sin^2\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{12}$, δ
from F. Capozzi et al., arXiv:1312.2878v2 (May 5, 2014).

$$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

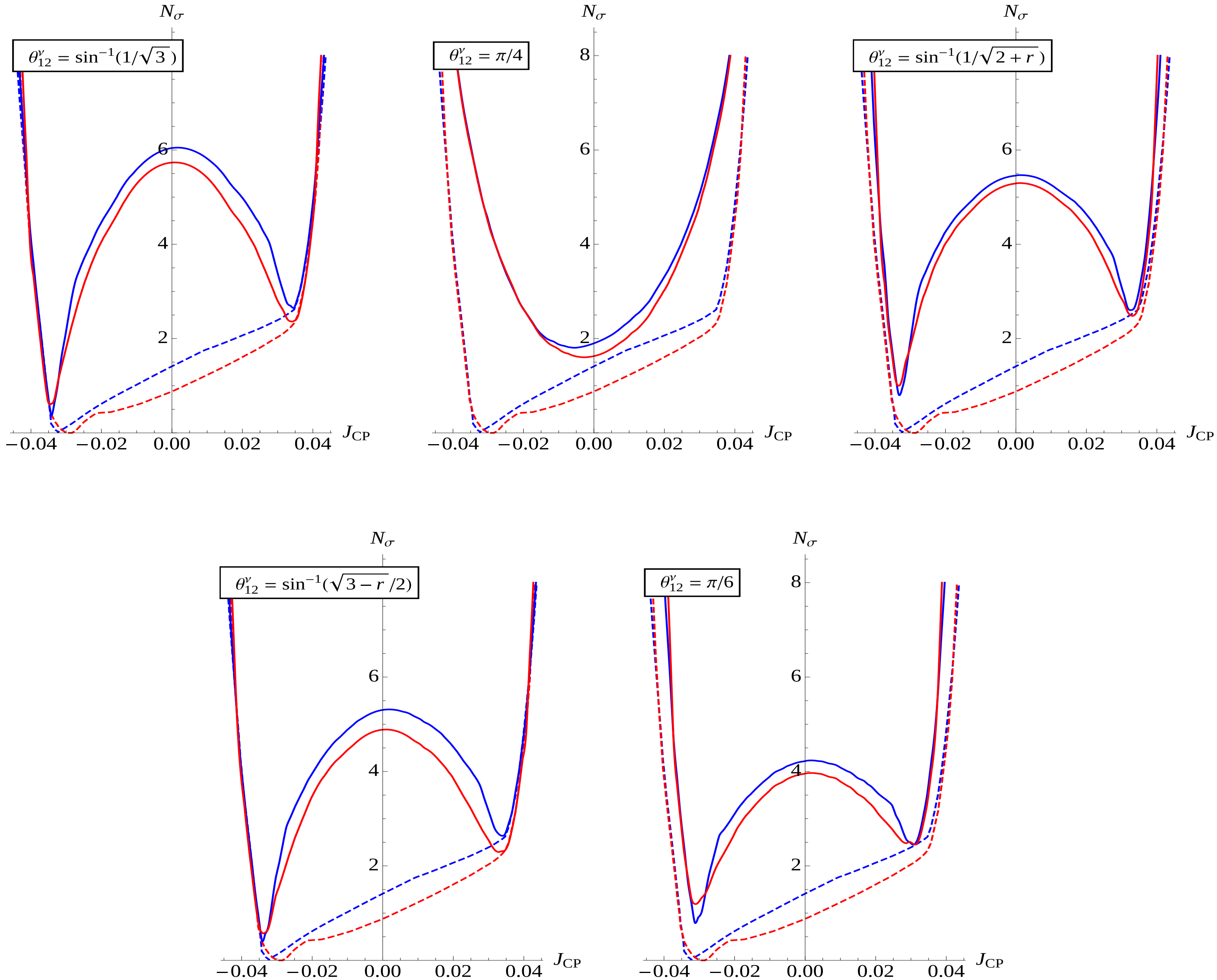
nσ confidence level interval of values of $\cos\delta$**:**

$$L(\cos\delta) \geq L(\chi^2_{\rm min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J = 0$ **excluded at** 5σ**,** 4σ**,** 4σ**,** 3σ **confidence level.**

At 3σ**:** $0.020 \leq |J_{\rm CP}| \leq 0.039$**.**

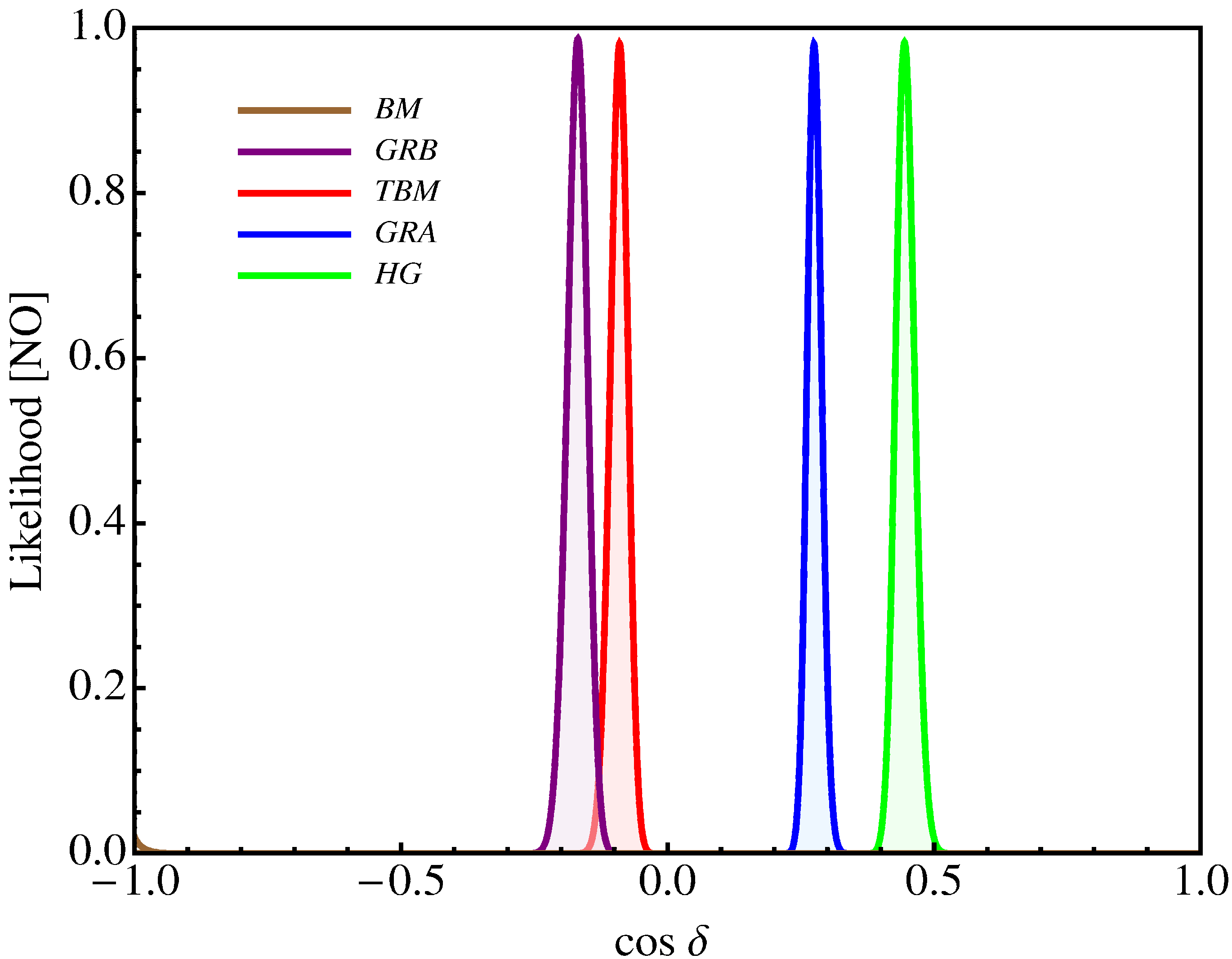
BM (LC), b.f.v.: $J_{\rm CP} = 0$**;**
at 3σ**:** $-0.026\ (-0.025) \leq J_{\rm CP} \leq 0.021\ (0.023)$ **for NO (IO) neutrino mass spectrum.**

Prospective precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

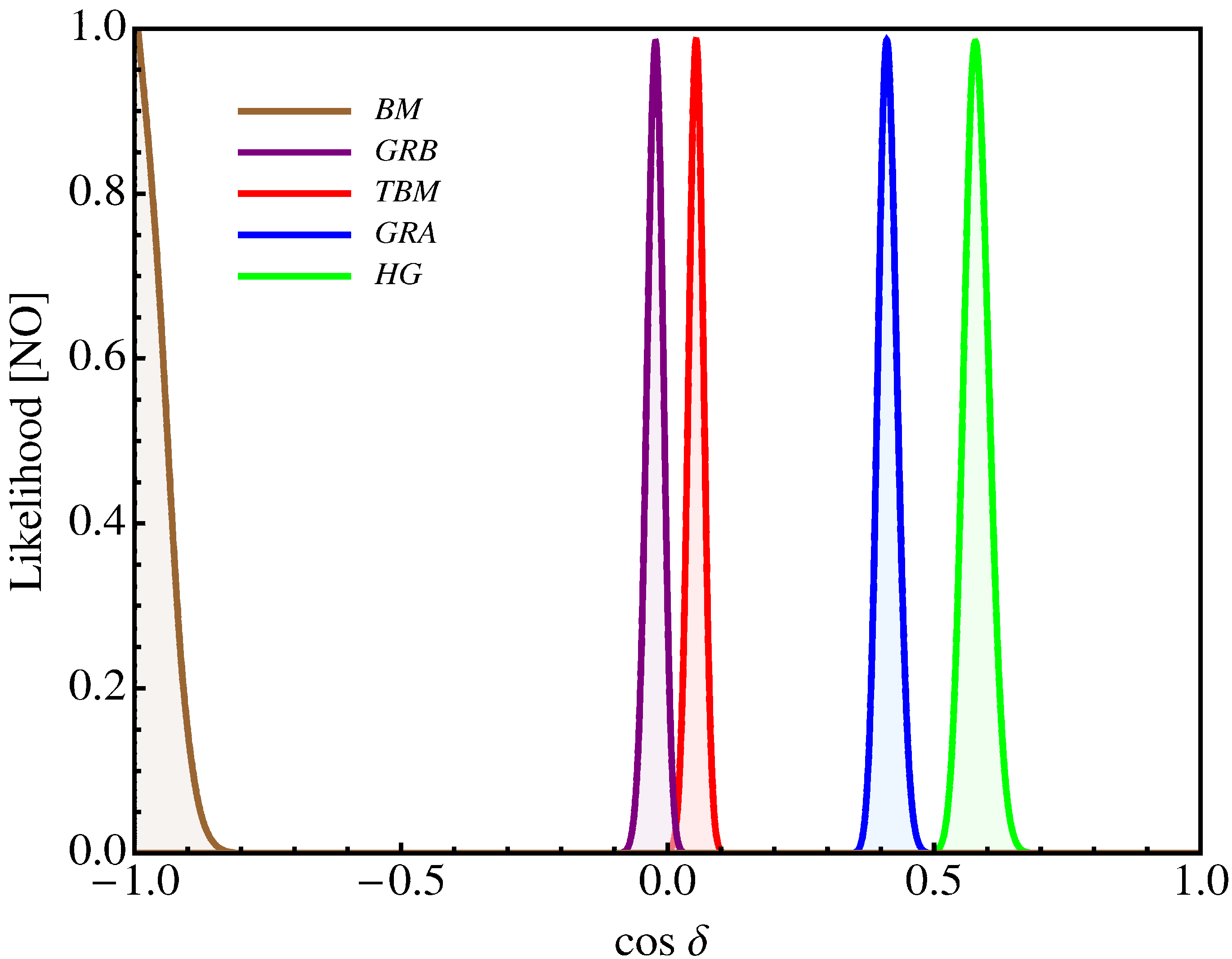
$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 5\%$ **(T2K, NOνA combined).**



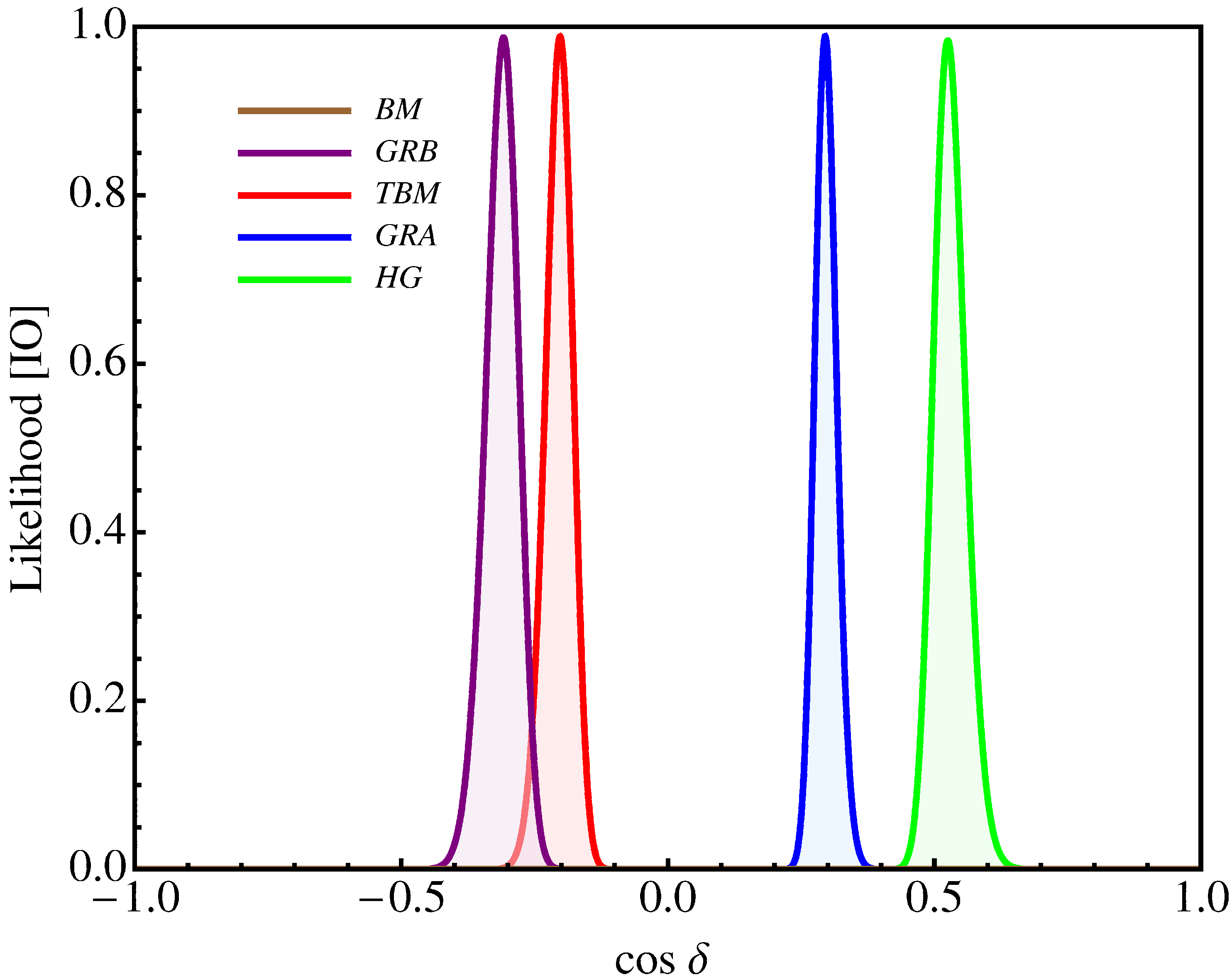
I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

b.f.v. of $\sin^2\theta_{ij}$ (Capozzi et al., 2014) + the prospective precision used.



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

The same, but for $\sin^2\theta_{12} = 0.33$ **(the BM prediction dependence on** $\sin^2\theta_{12}$**).**



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$\sin^2\theta_{23} = 0.557$ **(b.f.v.: C. Gonzales-Garcia et al., 2014, IO case).**

The predictions obtained for $\cos\delta$ are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

J. Gehrlein *et al.*, **"An $SU(5) \times A_5$ Golden Ratio Flavour Model", arXiv:1410.2095;**

I. Girardi *at al.*, **"Generalised Geometrical CP Violation in a T' Lepton Flavour Model", arXiv:1312.1966, JHEP 1402 (2014) 050.**

T' model of lepton flavour: U_{TBM}, $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.

- ν_j - Majorana particles.

- Diagonalisation of M_ν: $U_{\text{TBM}}\Phi$, $\Phi = \text{diag}(1, 1, 1(i))$

- U_{TBM} "corrected" by

$U_{\text{lep}}^{\dagger} Q = R_{12}(\theta^{\ell}_{12})\, R_{23}(\theta^{\ell}_{23}) Q\,, \quad Q = \text{diag}(1, e^{i\phi}, 1)$

T' model of lepton flavour: U_{TBM} , $\delta \cong 3\pi/2$ or $\pi/2$.

• T': double covering of A_4 (tetrahedral symmetry group).

• T': $\mathbf{1}$, $1'$, $1''$; $\mathbf{2}$, $2'$, $2''$; $\mathbf{3}$.

• T' model: $\psi_{eL}(x)$,$\psi_{\mu L}(x)$,$\psi_{\tau L}(x)$ - triplet of T';
$e_R(x)$,$\mu_R(x)$ - a doublet, $\tau_R(x)$ - a singlet, of T';
$\nu_{eR}(x)$,$\nu_{\mu R}(x)$,$\nu_{\tau R}(x)$ - a triplet of T';
the Higgs doublets $H_u(x)$, $H_d(x)$ - singlets of T'.

• The discrete symmetries of the model are $T' \times H_{\text{CP}} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$, the Z_n factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

Predictions of the T' Model

- $m_{1,2,3}$ determined by 2 real parameters + Φ^2:

$$\text{NO spectrum A}: \ (m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3} \text{ e}$$

$$\text{NO spectrum B}: \ (m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3} \text{ e}$$

$$\text{IO spectrum}: \ (m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3}$$

$$\text{NO A}: \quad \sum_{j=1}^{3} m_j = 6.29 \times 10^{-2} \text{ eV},$$

$$\text{NO B}: \quad \sum_{j=1}^{3} m_j = 6.52 \times 10^{-2} \text{ eV},$$

$$\text{IO}: \quad \sum_{j=1}^{3} m_j = 12.11 \times 10^{-2} \text{ eV},$$

- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

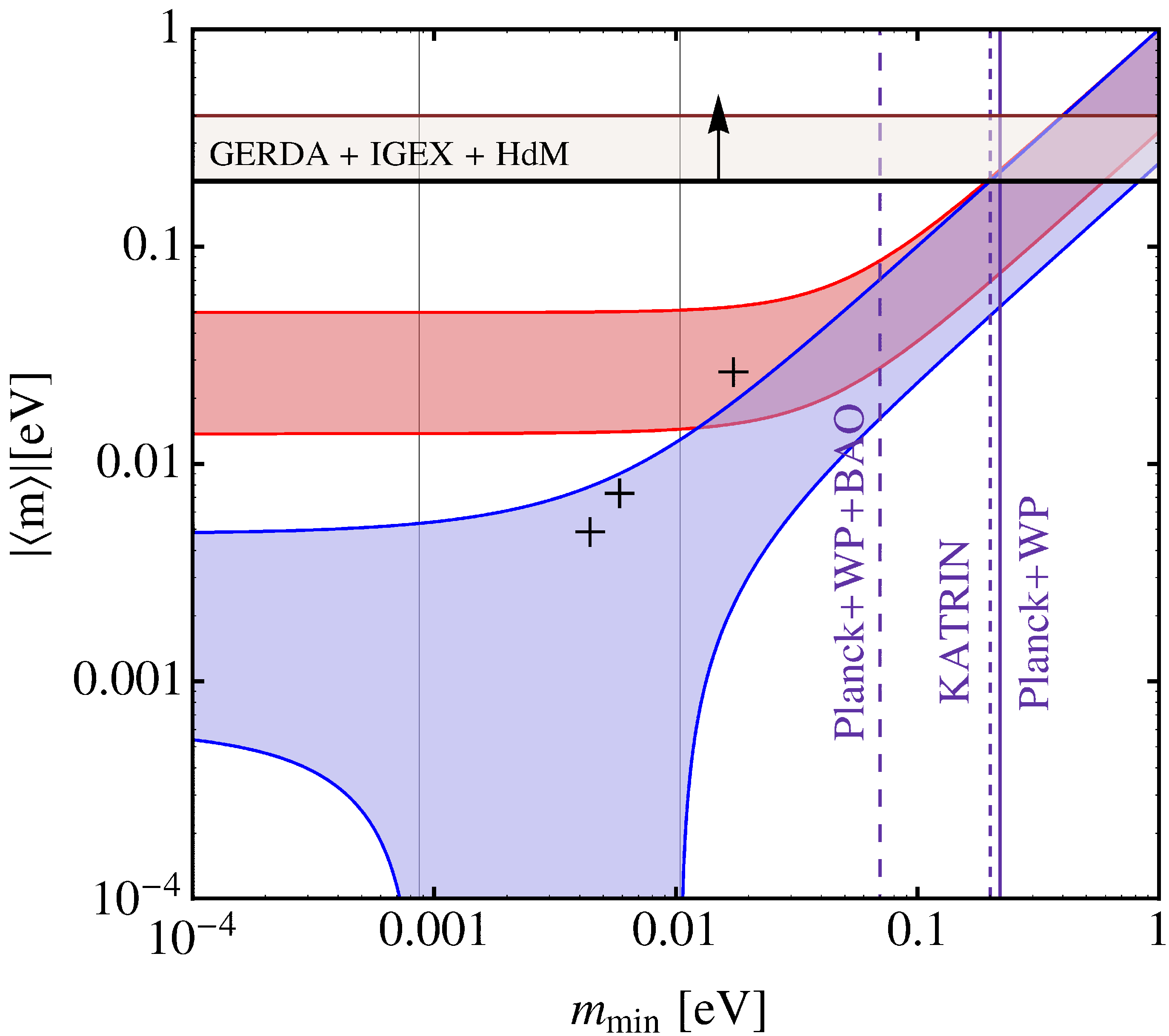
Given the values of $\theta_{12}, \theta_{23}, \theta_{13}$, $\delta, \alpha_{21}, \alpha_{31}$ are predicted:

$\delta \cong 3\pi/2$ (266°) (or $\pi/2$ (94°));

NO A: $\alpha_{21} \cong +47.0^\circ$ (or -47.0°) $(+2\pi)$,

$\alpha_{31} \cong -23.8^\circ$ (or $+23.8^\circ$) $(+2\pi)$.

The model is falsyfiable.



1
0.1
0.01
0.001
10^{-4}
$|\langle m \rangle|$[eV]
10^{-4}
0.001
0.01
0.1
1
$m_{\rm min}$ [eV]
GERDA + IGEX + HdM
Planck+WP+BAO
KATRIN
Planck+WP

The Nature of Massive Neutrinos

Historical Comment

Pontecorvo, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,, \quad m_1 \neq m_2 > 0\,, \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$ - Majorana, maximal mixing .

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos\theta_C + \Psi_{2L} \sin\theta_C\,,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin\theta_C + \Psi_{2L} \cos\theta_C\,,$$

$\Psi_{1,2}$ - Dirac (composite), θ_C- the Cabbibo angle .

Determining the Nature of Massive Neutrinos

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} \ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} \ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21}, \ \alpha_{31}$

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U^{\dagger}_{jl}$$

$$U = VP \ : \ \ P_j e^{-i(E_j t - p_j x)} P^*_j = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j.

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$**,** $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of Δm^2_{atm} and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

If ν_j- Majorana particles, $U_{\rm PMNS}$ contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \to \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \ (\mathrm{A}, \mathrm{Z}) \to \mu^+ + \ (\mathrm{A}, \mathrm{Z} - 2)$$

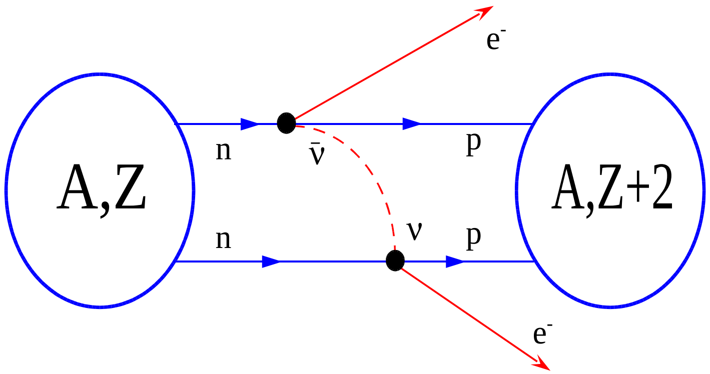
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\mathrm{A}, \mathrm{Z}) \to (\mathrm{A}, \mathrm{Z} + 2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

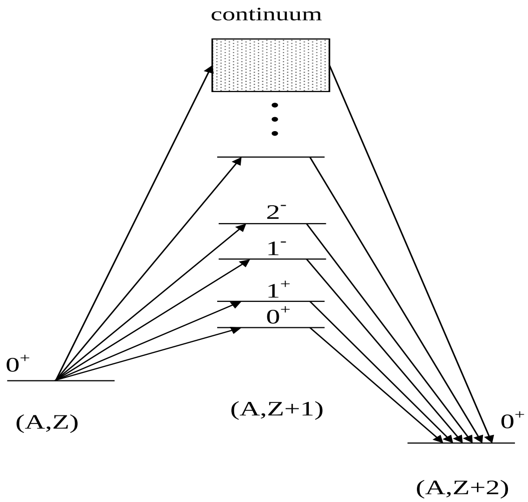
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \to uue^-e^-(\overline{\nu}_e\overline{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}$–Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha_{31}}\right|,\quad \theta_{12} \equiv \theta_{\odot},\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left|\sqrt{\Delta m^2_{\odot}}\, \sin^2\theta_{12} e^{i\alpha} + \sqrt{\Delta m^2_{31}}\, \sin^2\theta_{13} e^{i\beta_M}\right| , \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\, \left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right| , \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|<m>| \cong m \left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right| , \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

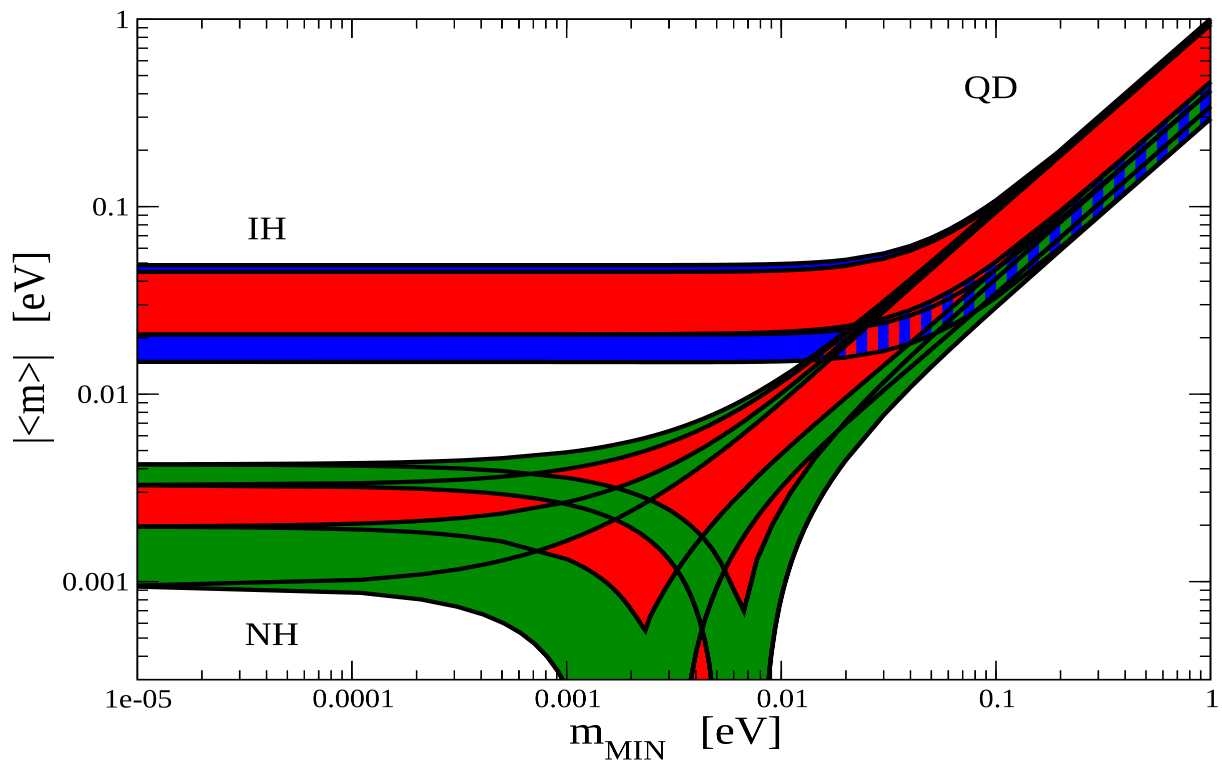
$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta_M \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$$|<m>| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m^2_{13}} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV}, \quad \text{IH};$$

$$m \cos 2\theta_{12} \lesssim |<m>| \lesssim m,\ m \gtrsim 0.10 \text{ eV}, \quad \text{QD} .$$



S. Pascoli, PDG, 2012

$\sin^2\theta_{13} = 0.0236 \pm 0.0042$; $\delta = 0$.

$1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$, $1\sigma(|\Delta m^2_{31(23)}|) = 3\%$.

From G.L. Fogli *et al.*, arXiv:1205.5254v3

$2\sigma(|<m>|\ )$ used.

Best sensitivity: GERDA (^{76}Ge), EXO (^{136}Xe), KamLAND-ZEN (^{136}Xe).

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\!<\!m\!>\!| = (0.1 - 0.9)$ eV (99.73% C.L.); b.f.v.: $|\!<\!m\!>\!| = 0.33$ eV.

IGEX ^{76}Ge: $|\!<\!m\!>\!| < (0.33 - 1.35)$ eV (90% C.L.).

Recent data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|\!<\!m\!>\!| <$(0.45–0.96) eV, $|\!<\!m\!>\!| <$(0.18–0.64) eV (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$T(^{76}\mathrm{Ge}) = 2.23^{+0.44}_{0.31} \times 10^{25} \text{ yr at } 90\% \text{ C.L.}$$

Results from 2012-2013:

$$T(^{136}\mathrm{Xe}) > 1.6 \times 10^{25} \text{yr at } 90\% \text{ C.L., EXO}$$

$$T(^{136}\mathrm{Xe}) > 1.9 \times 10^{25} \text{yr at } 90\% \text{ C.L., KamLAND} - \text{Zen}$$

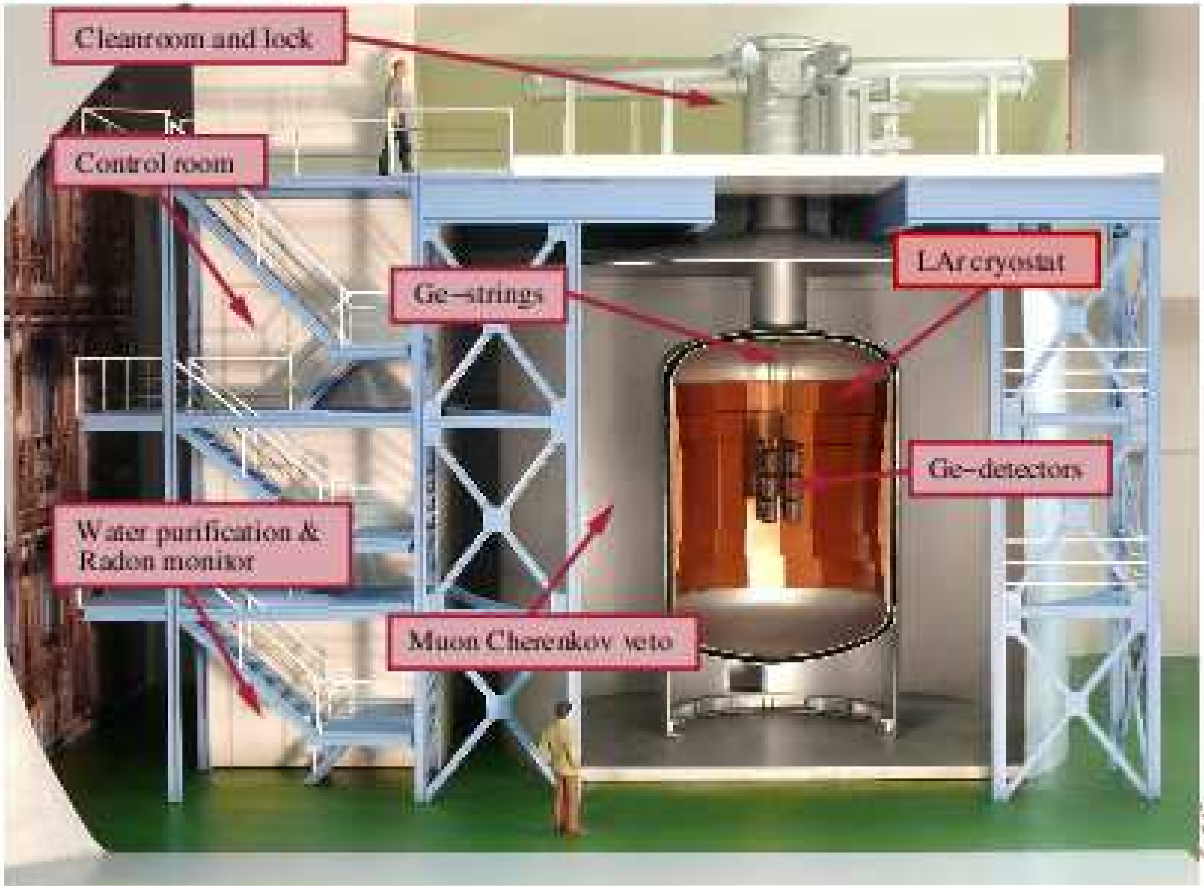
$$T(^{76}\mathrm{Ge}) > 2.1 \times 10^{25} \text{yr at } 90\% \text{ C.L., GERDA.}$$

$$T(^{76}\mathrm{Ge}) > 3.0 \times 10^{25} \text{yr at } 90\% \text{ C.L., GERDA} + \text{IGEX} + \text{HdM}$$

Large number of experiments: $|<m>| \sim$ (0.01-0.05) eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
KamLAND-ZEN - ^{136}Xe;
EXO - ^{136}Xe;
SNO+ - ^{130}Te;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca;
SuperNEMO - ^{82}Se,...;
MAJORANA - ^{76}Ge;
COBRA - ^{116}Cd;
MOON - ^{100}Mo.

GERDA: Experimental Setup



Majorana CPV Phases and $|<m>|$

CPV can be established provided

- $|<m>|$ measured with $\Delta \lesssim 15\%$;

- Δm^2_{atm} (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

- $\xi \lesssim 1.5$;

- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

- $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

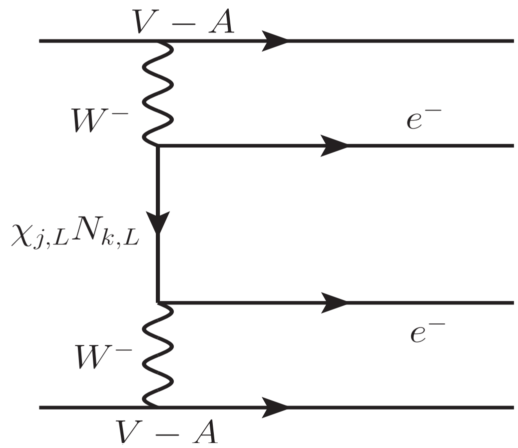
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

Different Mechanisms of $(\beta\beta)_{0\nu}$-Decay



Light Majorana Neutrino Exchange

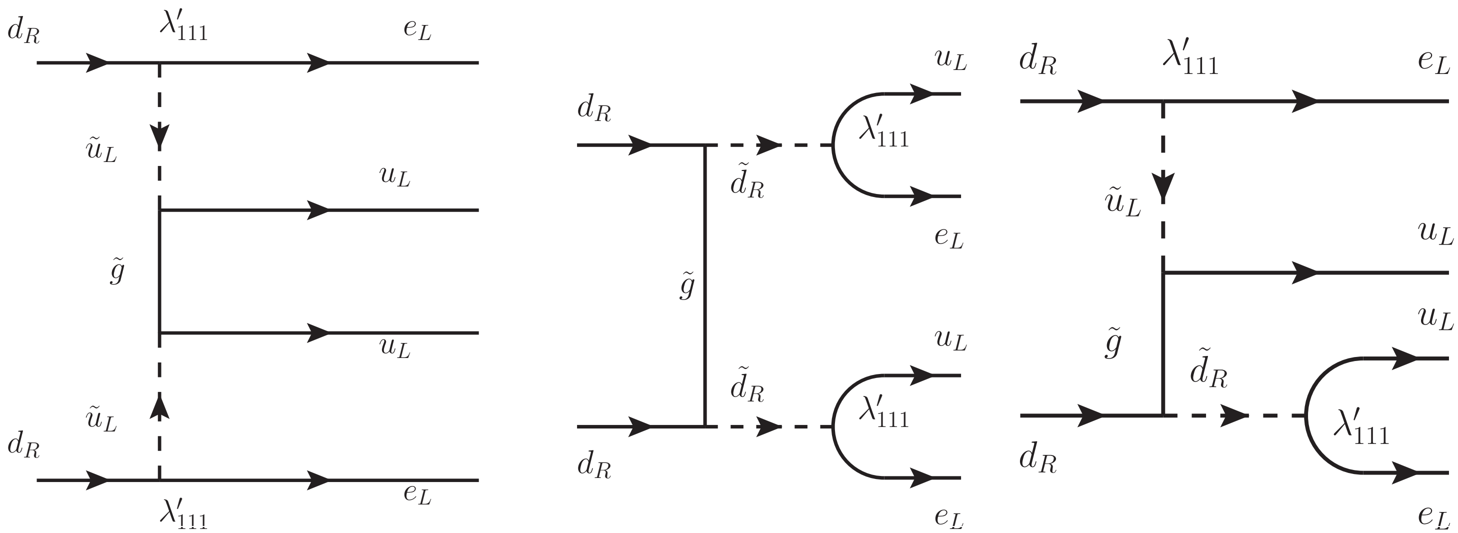
$$\eta_\nu = \frac{<m>}{m_e} \,.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k, $M_k \gtrsim 10$ GeV:

$\eta_N^L \;=\; \Sigma_k^{heavy} \; U_{ek}^2 \frac{m_p}{M_k}$, m_p - proton mass , U_{ek} - CPV .

SUSY Models with R-Parity Non-conservation



$$\mathcal{L}_{R_p} = \lambda'_{111}\left[(\bar{u}_L\ \bar{d}_L)e^c_R - \nu^c_{eR}\tilde{d}_R + (\bar{e}_L\ \bar{\nu}_{eL})d_R\tilde{u}^*_L - \tilde{d}^*_L\right.$$

$$\left. + \ (\bar{u}_L\ \bar{d}_L)d_R\tilde{e}^*_L - \tilde{\nu}^*_{eL}\right] + h.c.$$

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$-decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, "Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay", arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Meroni, S.T.P. and F. Simkovic, "Multiple CP Non-conserving Mechanisms of bb0nu-Decay and Nuclei with Largely Different Nuclear Matrix Elements", (arXiv:1212.1331, JHEP **1302** (2013) 025.

Earlier studies include:
A. Halprin, S.T.P., S.P. Rosen, "Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. 125B (1983) 335).

Origins of Dirac and Majorana Massive Neutrinos

• Massive Dirac Neutrinos: $U(1)$, Conserved (Additive) Charge, e.g., L.

• Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}^{\nu}_{m}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}^{\nu}_{m}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

Mass Term: any by-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}^{\nu}_{m}(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}^{\nu}_{m}(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

- Dirac Neutrinos: Dirac Mass Term, requires $\nu_R(x)$ - $SU(2)_L$ singlet RH ν fields

$$\mathcal{L}^{\nu}_{D}(x) = -\ \overline{\nu_{l'R}}(x)\ M_{Dl'l}\ \nu_{lL}(x) + h.c.\ ,\ \ M_D - complex$$

- $\mathcal{L}^{\nu}_{D}(x)$ conserves L: $L = const.$

$$M_D = V M_D^{diag} W^{\dagger}\,,\ \ V, U - \text{unitary}\ (bi - unitary\ transformation)\,,\ \ W \equiv U_{\text{PMNS}}$$

- ST + 3 $\nu_R(x)$ - RH ν fields: $n = 3$

$$\begin{aligned} \mathcal{L}_{\text{Y}}(x) &= Y^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,, \\ M_D &= \frac{v}{\sqrt{2}}Y^{\nu}\,,\quad v = 246\ \text{GeV}\,. \end{aligned}$$

No explanation why $m(\nu_j) <<< m_l, m_q$.

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes $\mu^+ \to e^+ + \gamma$ decay, $\mu^- \to e^- + e^+ + e^-$ decay, $\tau^- \to e^- + \gamma$ decay, etc. are allowed.

However, they are predicted to proceed with unobservable rates:

$$BR(\mu \to e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U^*_{\mu j} \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},$$

$$M_W \cong 80 \text{ GeV}, \text{ the W}^\pm - \text{mass}$$

S.T.P., 1976

"New Physics": $\nu_l \to \nu_{l'}$, $\bar{\nu}_l \to \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$ oscillations.

- Majorana ν_j: Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$

$$\mathcal{L}^{\nu}_{M}(x) = \frac{1}{2}\, \nu^{\mathsf{T}}_{l'L}(x)\; C^{-1}\; M_{l'l}\; \nu_{lL}(x) + h.c.\;,\;\; C^{-1}\gamma_\alpha C = -\gamma^{\mathsf{T}}_\alpha$$

- If $M_{l'l} \neq 0$, $L_l \neq const.$, $L \neq const.$, $n = 3$

- $\nu_{lL}(x)-$fermions: $M = M^{\mathsf{T}}$, complex.

$$M^{diag} = U^{\mathsf{T}} M U\,,\; U - \text{unitary (congruent transformation)};\; U \equiv U_{\text{PMNS}}$$

$$\nu_j \equiv \chi_j(x) = U^{\dagger}_{jl}\nu_{lL}(x) + U^{*}_{jl}\nu^{c}_{lR} = C\;(\overline{\chi_j}(x))^{\mathsf{T}}\,,\quad m_j \neq 0\,,\; j = 1, 2, 3$$

CP-invariance: $M^* = M$, M - real, symmetric.

$M^{diag} = (m'_1, m'_2, m'_3)$: $m'_j = \rho_j m_j$, $m_j \geq 0$, $\rho_j = \pm 1$

χ_j: $m_j \geq 0$: $\eta_{CP}(\chi_j) = i\rho_j$

$\mathcal{L}^{\nu}_{M}(x)$ not possible in the ST: requires New Physics Beyond the ST

$(\beta\beta)_{0\nu}$-decay is allowed; typically also $BR(\mu \to e + \gamma)$, $BR(\mu \to 3e)$, $CR(\mu^- + \mathcal{N} \to e^- + \mathcal{N})$ can be "large", i.e., in the range of sensitivity of ongoing (MEG) and future planned experiments.

• Majorana ν_j: Dirac+Majorana Mass Term; requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$:

$$\mathcal{L}^{\nu}_{D+M}(x) = - \overline{\nu_{l'R}}(x)\, M_{Dl'l}\, \nu_{lL}(x) + \frac{1}{2}\, \nu^{\mathsf{T}}_{l'L}(x)\, C^{-1}\, M^{LL}_{l'l}\, \nu_{lL}(x) + \frac{1}{2}\, \nu^{\mathsf{T}}_{l'R}(x)\, C^{-1}\, (M^{RR})^{\dagger}_{l'l}\, \nu_{lR}(x) -$$

$$M = \begin{pmatrix} M^{LL} & M_D \\ M_D^T & M^{RR} \end{pmatrix} = M^T \quad \left((M^{LL})^T = M^{LL},\ (M^{RR})^T = M^{RR)}\right)$$

• If $M_{Dl'l} \neq 0$ and $M^{LL}_{l'l} \neq 0$ and/or $M^{RR}_{l'l} \neq 0$: $L_l \neq const.$, $L \neq const.$; $n = 6\ (> 3)$

• $M = M^{\mathsf{T}}$, complex.

$$M^{diag} = W^{\mathsf{T}} M W\,,\ W - \text{unitary},\ 6 \times 6\,;\quad W^T \equiv (U^T \quad V^T)\,;\ U \equiv U_{\text{PMNS}}:\ 3 \times 6\,.$$

$$\nu_{lL}(x) = \sum_{j=1}^{6} U_{lj} \chi_j(x)\,,\quad \chi_j(x) - \text{Majorana}\ ,\quad m_j \neq 0\,,\quad l = e, \mu, \tau\,;$$

$$\nu^C_{lL}(x) \equiv C\ (\overline{\nu_{lR}}(x))^{\mathsf{T}} = \sum_{j=1}^{6} V_{lj} \chi_j(x)\,,\quad \nu^C_{lL}(x):\ \text{sterile antineutrino}$$

$\mathcal{L}^{\nu}_{D+M}(x)$ possible in the ST + ν_{lR}: $M^{LL} = 0$

$(\beta\beta)_{0\nu}$-decay is allowed;
phenomenology depends on the relative magnitude of M_D and M^{RR}.

Dirac - Majorana Relation (if any...)

Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$, can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}_M^{\nu}(x) = -\frac{1}{2}\,\overline{\nu^c_{l'R}}(x)\; M_{l'l}\;\nu_{lL}(x) + h.c.\;,\;\; \nu^c_{l'R} \equiv C\;(\overline{\nu_{l'L}}(x))^{\mathsf{T}}$$

$\mathcal{L}_M^{\nu}(x)$ conserves, e.g. $L' = L_e - L_\mu - L_\tau$ if only $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$

S.T.P., 1982

- Dirac ν, Ψ, is equivalent to two Majorana ν's, $\chi_{1,2}$, having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,,\quad m_1 = m_2 = m_D > 0\,, \eta_{jCP} = i\rho_j\,, \rho_1 = -\rho_2\;\; (C\;(\overline{\chi_j})^{\mathsf{T}} = \rho_j\chi_j)$$

Example ZKM ν : $\nu_{eL}(x) = \Psi_L = \dfrac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}$, $\nu_{\mu L}(x) = \Psi_L^C = \dfrac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$

- Pseudo-Dirac Neutrino: the symmetry of $\mathcal{L}_M^{\nu}(x)$ is not a symmetry of $\mathcal{L}_{tot}(x)$

Suppose: $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$, and to "leading order" $m_1 = m_2$, but due to "higher order" corrections $m_1 \neq m_2$, $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects $\sim \Delta m$

- Suppose: $m_1 = m_2$, $\rho_1 = -\rho_2$, but $\chi_{1,2}$ are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L}\cos\phi + \chi_{2L}\sin\phi = \Psi_L\cos\phi' + \Psi_L^C\sin\phi'$$

All Majorana effects are $\sim m_D\cos\phi'\sin\phi'$

In the case of conserved $L' = L_e - L_\mu - L_\tau$:

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$, $\theta_{13} = 0$, $\tan\theta_{23} = M_{e\tau}/M_{e\mu}$,

$m_3 = 0$ - spectrum with IH, $m_1 = m_2$, $\chi_{1,2}$ - equivalent to one Dirac ν, Ψ .

Adding L'-breaking term, e.g. M_{ee}, $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$, leads to $m_1 \neq m_2$ compatible with $\Delta m^2_\odot$.

Origin of Neutrino Masses and Mixing

The Seesaw Mechanisms of Neutrino Mass Generation

- Explain the smallness of $\nu-$masses.

- Through leptogenesis theory link the $\nu-$mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH νs' (heavy).

Type II seesaw mechanism: $\mathbf{H(x)}$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $\mathbf{T(x)}$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$-decay, LFV processes, etc.) and New Physics at LHC.

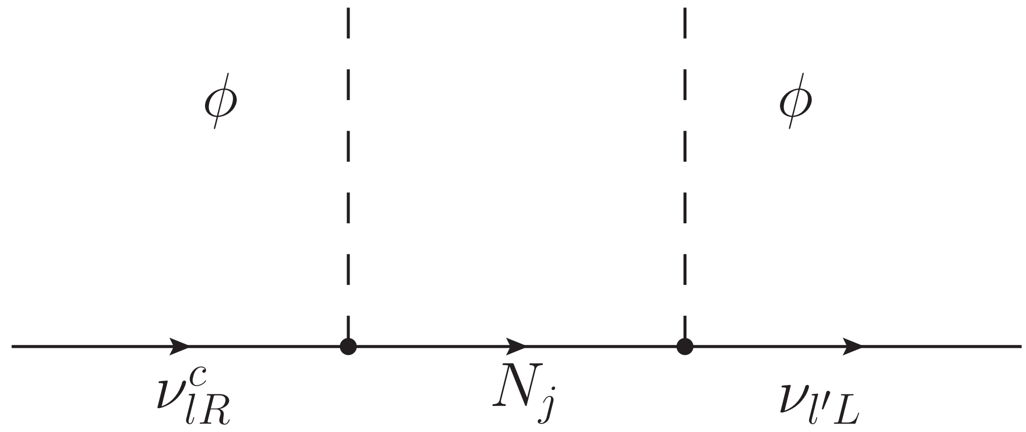
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D = vY^{\nu}/\sqrt{2}| << |M^{RR}|$.
- Diagonalising M^{RR}: N_j - heavy Majorana neutrinos, $M_j \sim$TeV; or $(10^9 - 10^{13})$ GeV in GUTs.

For sufficiently large M_j, Majorana mass term for $\nu_{lL}(x)$:

$$M_\nu \cong v_u^2 \,(Y^\nu)^T \, M_j^{-1} \, Y^\nu = U^*_{\text{PMNS}} \, m_\nu^{\text{diag}} \, U^\dagger_{\text{PMNS}} \,.$$

$v_u Y^\nu = M_D$, $M_D \sim 1$ GeV, $M_j = 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.



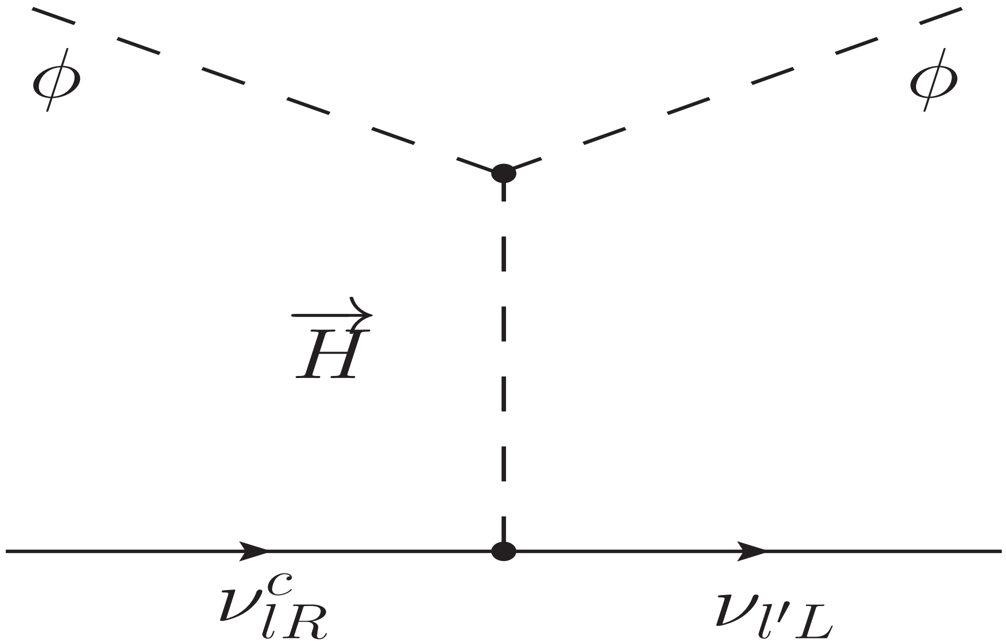
• $\nu_{l'R}(x)$: Majorana mass term at "high scale" ($\sim$TeV; or $(10^9 - 10^{13})$ GeV in $SO(10)$ GUT)

$$\mathcal{L}^{\nu}_{M}(x) = +\,\frac{1}{2}\,\nu^{\mathsf{T}}_{l'R}(x)\;C^{-1}\;(M^{RR})^{\dagger}_{l'l}\;\nu_{lR}(x) + h.c. = -\,\frac{1}{2}\sum_j \bar{N}_j\,M_j\,N_j\;,$$

• Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{l'R}(x)$ involving $\Phi(x)$:

$$\begin{aligned}
\mathcal{L}_{\mathsf{Y}}(x) &= \bar{Y}^{\nu}_{l'l}\,\overline{\nu_{l'R}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,,\\
&= Y^{\nu}_{jl}\,\overline{N_{jR}}(x)\,\Phi^T(x)\,(i\tau_2)\,\psi_{lL}(x) + \text{h.c.}\,,\\
M_D &= \frac{v}{\sqrt{2}}\,Y^{\nu}\,,\quad v = 246\ \text{GeV}\,.
\end{aligned}$$

Type II Seesaw Mechanism

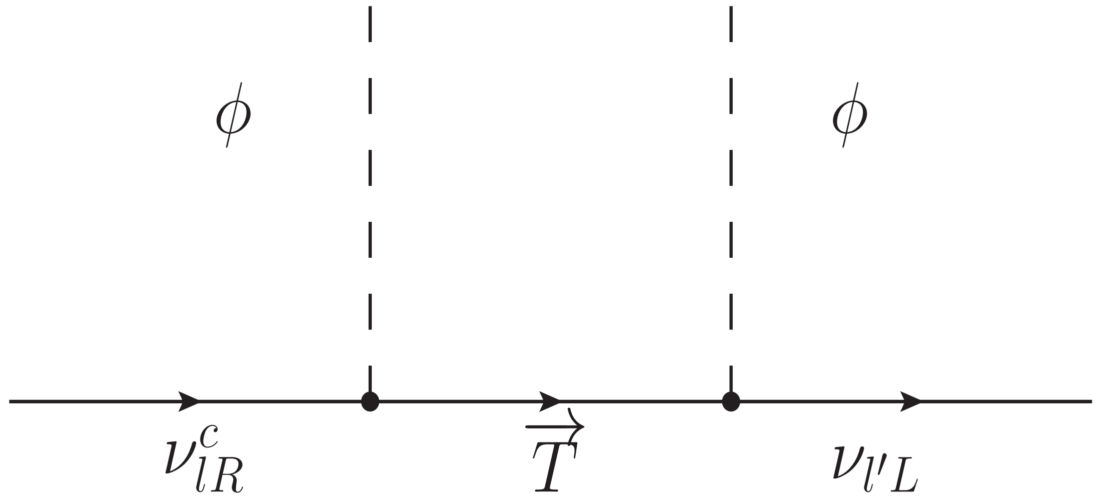


Due to I. Girardi

$$M_\nu \cong h\, v^2\, M_H^{-1} = U^*_{\text{PMNS}}\, m_\nu^{\text{diag}}\, U^\dagger_{\text{PMNS}}\ .$$

$h \sim 10^{-2}$, $v = 246$ GeV, $M_H \sim 10^{12}$ GeV: $M_\nu \sim 0.6$ eV.

Type III Seesaw Mechanism



$$M_\nu \cong v^2\ (Y_T)^T\, M_T^{-1}\, Y_T = U^*_{\mathrm{PMNS}}\ m_\nu^{\mathrm{diag}}\ U^\dagger_{\mathrm{PMNS}}\ .$$

$v\,Y_T \sim 1$ GeV, $M_T \sim 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.

TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos N_j at the TeV scale:

$m_\nu \simeq -\,M_D \widehat{M}_N^{-1} M_D^T$, $\widehat{M} = \text{diag}(M_1, M_2, M_3)$, $M_j \sim (100 - 1000)$ GeV.

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}}\,\bar{\ell}\,\gamma_\alpha\,(RV)_{\ell k}(1-\gamma_5)\,N_k\,W^\alpha + \text{h.c.}\,, \quad (RV)_{\ell k} \equiv U_{\ell 3+k}\,,$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w}\,\overline{\nu_{\ell L}}\,\gamma_\alpha\,(RV)_{\ell k}\,N_{kL}\,Z^\alpha + \text{h.c.}$$

- **All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:** $M_2 = M_1(1+z)$, $0 < z \ll 1$.

- **Only NH and IH ν mass spectra possible:** $\min(m_j) = 0$.

• **Requirements:** $|(RV)_{\ell k}|$ **"sizable"**
+ reproducing correctly the neutrino oscillation data:

$$
\begin{aligned}
|(RV)_{\ell 1}|^2 &= \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_3}{m_2+m_3}\left|U_{\ell 3} + i\sqrt{m_2/m_3}U_{\ell 2}\right|^2, \quad \text{NH},\\
|(RV)_{\ell 1}|^2 &= \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_2}{m_1+m_2}\left|U_{\ell 2} + i\sqrt{m_1/m_2}U_{\ell 1}\right|^2 \cong \frac{1}{4}\frac{y^2 v^2}{M_1^2}\left|U_{\ell 2} + iU_{\ell 1}\right|^2, \text{IH},\\
(RV)_{\ell 2} &= \pm i\,(RV)_{\ell 1}\sqrt{\frac{M_1}{M_2}}, \;\; \ell = e, \mu, \tau,
\end{aligned}
$$

y- **the maximum eigenvalue of** Y^ν, $v_u \simeq 174$ **GeV.**

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

4 parameters: M, z, y **and a phase** ω.

Low energy data:

$$
\begin{aligned}
|(RV)_{e1}|^2 &\lesssim 2\times 10^{-3},\\
|(RV)_{\mu 1}|^2 &\lesssim 0.8\times 10^{-3},\\
|(RV)_{\tau 1}|^2 &\lesssim 2.6\times 10^{-3}.
\end{aligned}
$$

S. Antusch et al., 2008

Observation of $N_{1,2}$ at LHC - problematic.

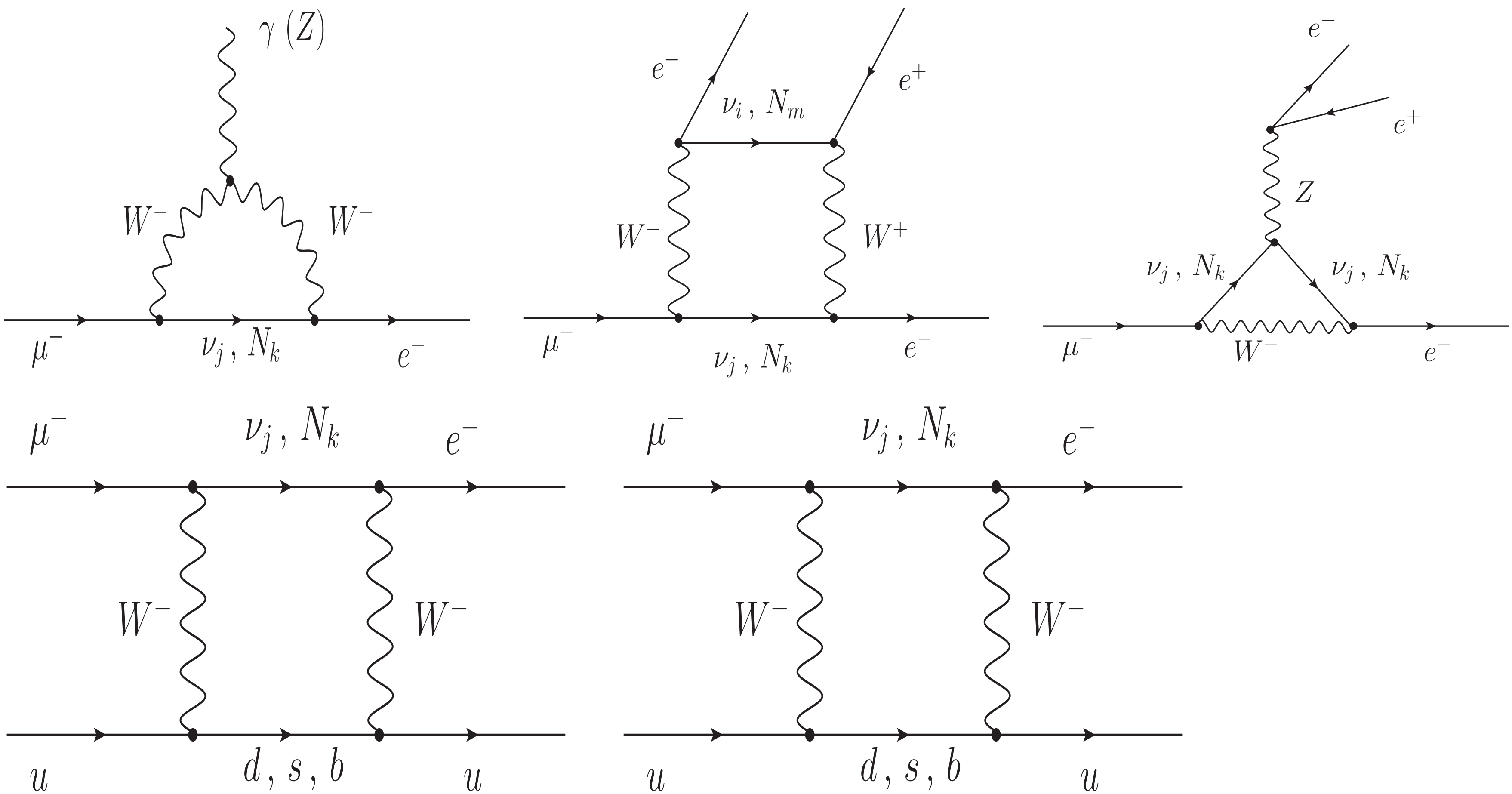
LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- + \mathcal{N} \to e^- + \mathcal{N}$ - the current limits:

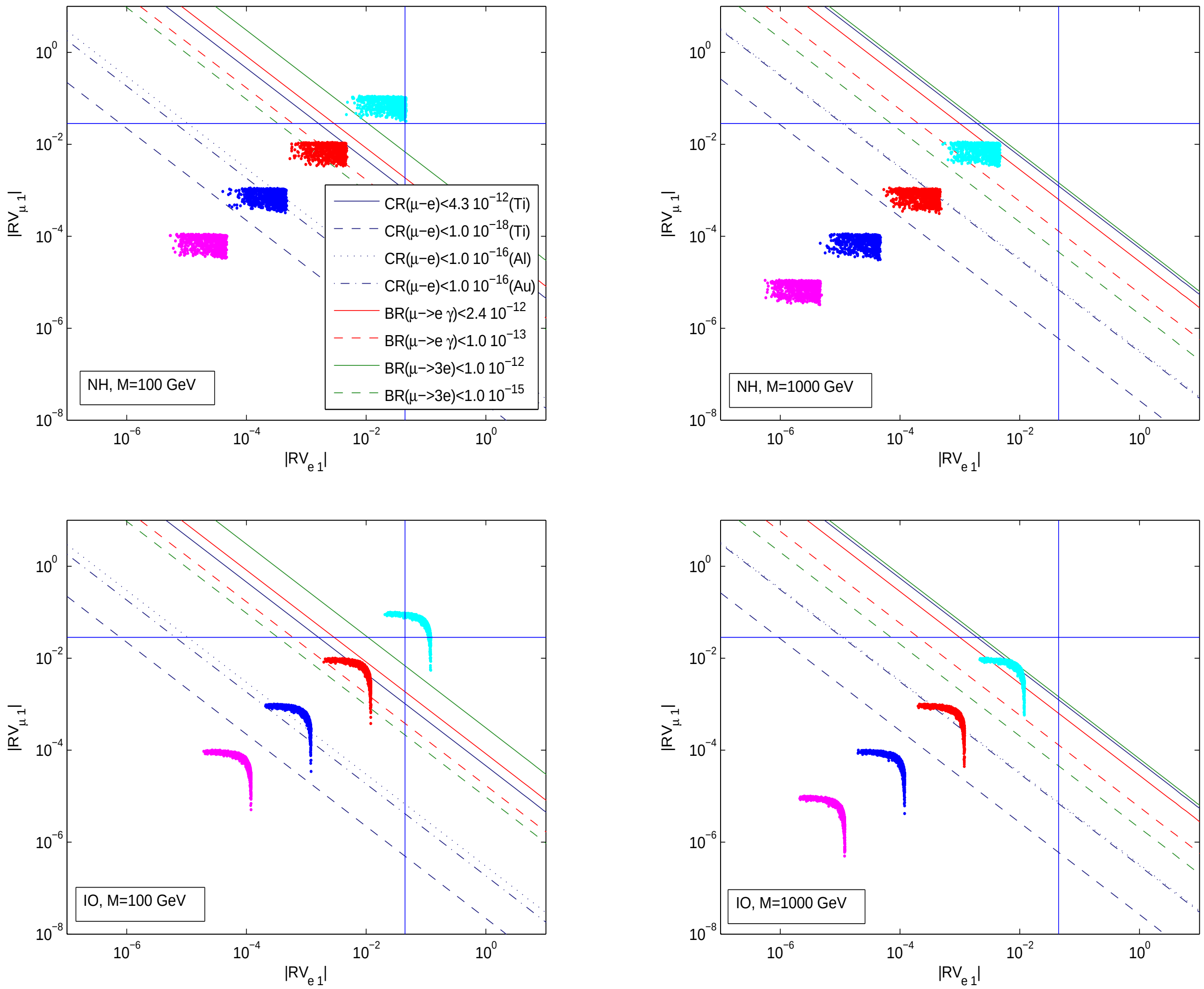
$$\text{BR}(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}\,,\ \text{MEG Collab., 2013};$$
$$\text{BR}(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12}\,,\ \text{SINDRUM Collab., 1988};$$
$$\text{CR}(\mu\text{Ti} \to e\text{Ti}) < 4.3 \times 10^{-12}\,,\text{SINDRUM Collab., 1993}.$$

Planned experiments on LFV processes.

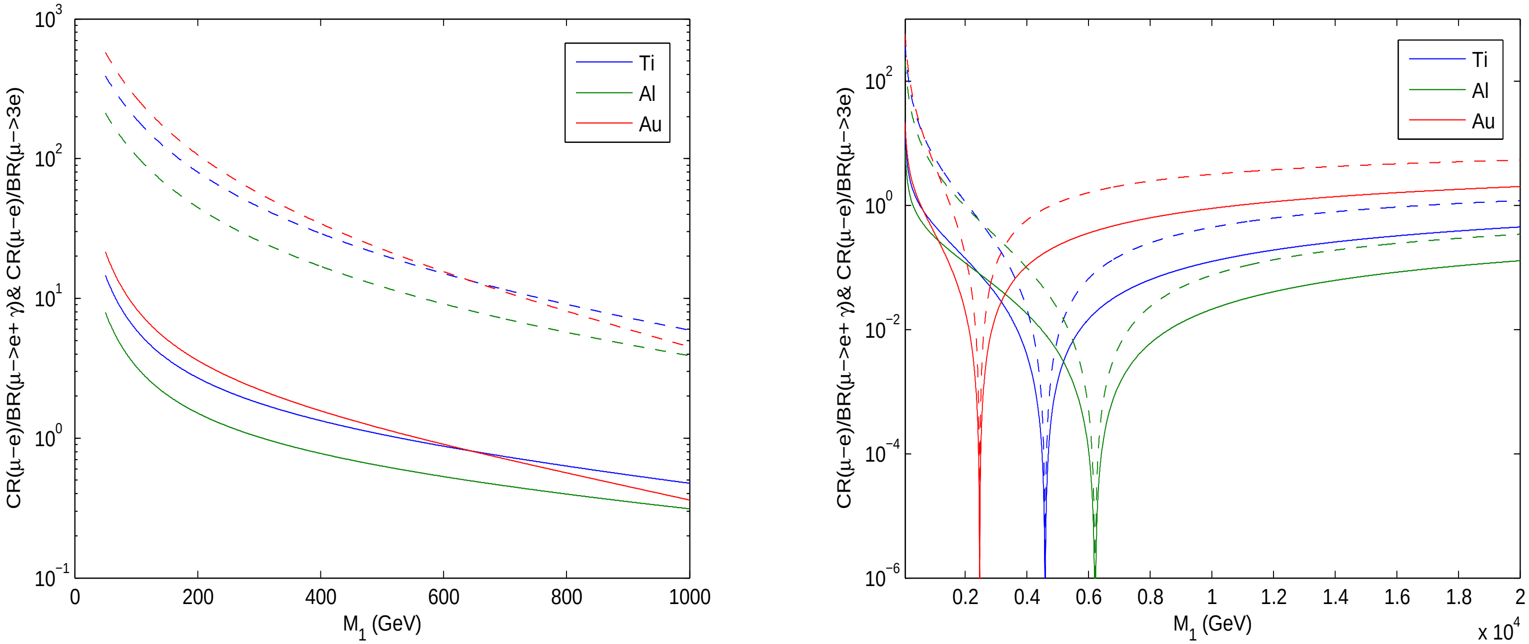
- **COMET (KEK), Mu2e (Fermilab):** $\text{CR}(\mu\,\text{Al} \to \text{e}\,\text{Al}) \approx 10^{-16}$.
- **PRISM/PRIME (KEK), Project-X (Fermilab):** $\text{CR}(\mu\,\text{Ti} \to \text{e}\,\text{Ti}) \approx 10^{-18}$.
- **MuSIC (Osaka Univ):** $\text{BR}(\mu^+ \to e^+e^-e^+) \approx 10^{-15}$.

LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- + \mathcal{N} \to e^- + \mathcal{N}$: can proceed with exchange of virtual N_j:





Current limits and potential sensitivity to $|(RV)_{e1}|$ and $|(RV)_{\mu 1}|$ from data on LFV processes for NH (upper panels) and IH (lower panels) spectra, for $M_1 =$ 100 (1000) GeV and, $i)$ $y = 0.0001$ (magenta pts), $ii)$ $y = 0.001$ (blue pts), $iii)$ $y = 0.01$ (red pts) and $iv)$ $y = 0.1$ (cyan pts).



The ratio of the $\mu - e$ relative conversion rate and the branching ratio of the i) $\mu \to e\gamma$ decay (solid lines), ii) $\mu \to 3e$ decay (dashed lines), versus the type I see-saw mass scale M_1, for three different nuclei: $^{48}_{22}$Ti (blue lines), $^{27}_{13}$Al (green lines) and $^{197}_{79}$Au (red lines).

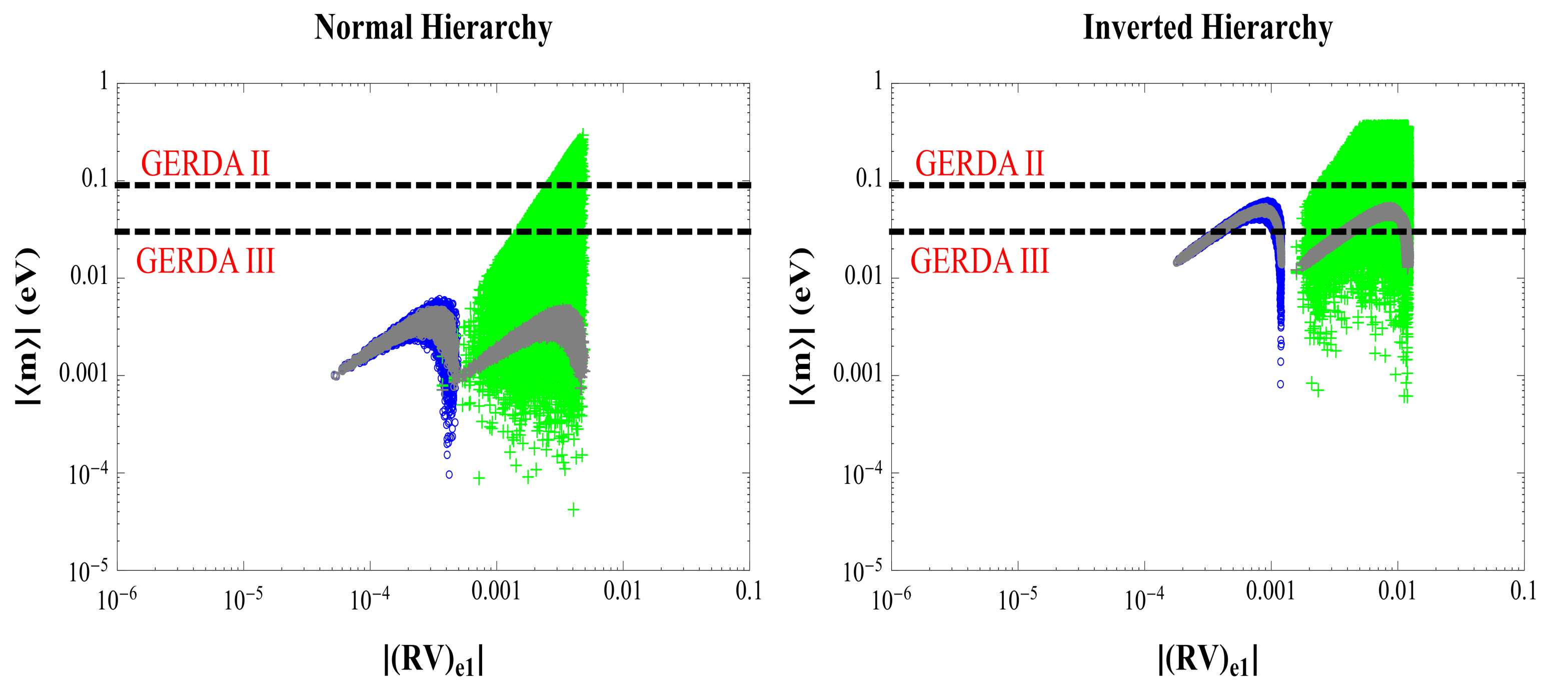
The exchange of virtual N_j gives a contribution to $|<m>|$:

$$|<m>| \;\cong \left|\sum_i (U_{PMNS})^2_{ei}\, m_i - \sum_k \, f(A, M_k)\,(RV)^2_{ek}\,\frac{(0.9\ \text{GeV})^2}{M_k}\right|\,,$$

$$f(A, M_k) \cong f(A)\,.$$

For, e.g., ^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te and ^{136}Xe, the function $f(A)$ takes the values $f(A) \cong$ 0.033, 0.079, 0.073, 0.085 and 0.068, respectively.

- **The Predictions for $|<m>|$ can be modified considerably.**



$|<m>|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and $i)$ $y = 0.001$ (blue), $ii)$ $y = 0.01$ (green). The gray markers correspond to $|<m>^{\rm std}| = |\sum_i (U_{PMNS})^2_{ei} m_i|$.

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

Instead of Conclusions

We are at the beginning of the Road...

The next 5, 10, 15,... years will be very exciting in Neutrino (Lepton) Physics!