# Petrov type I condition and Rindler Fluid

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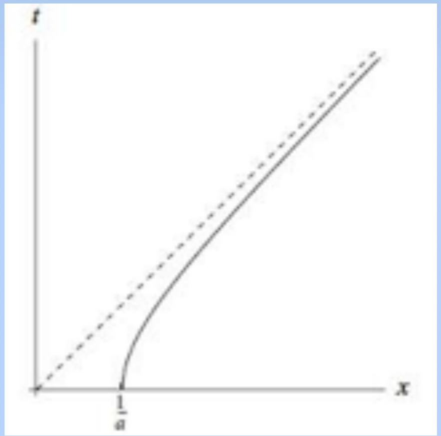
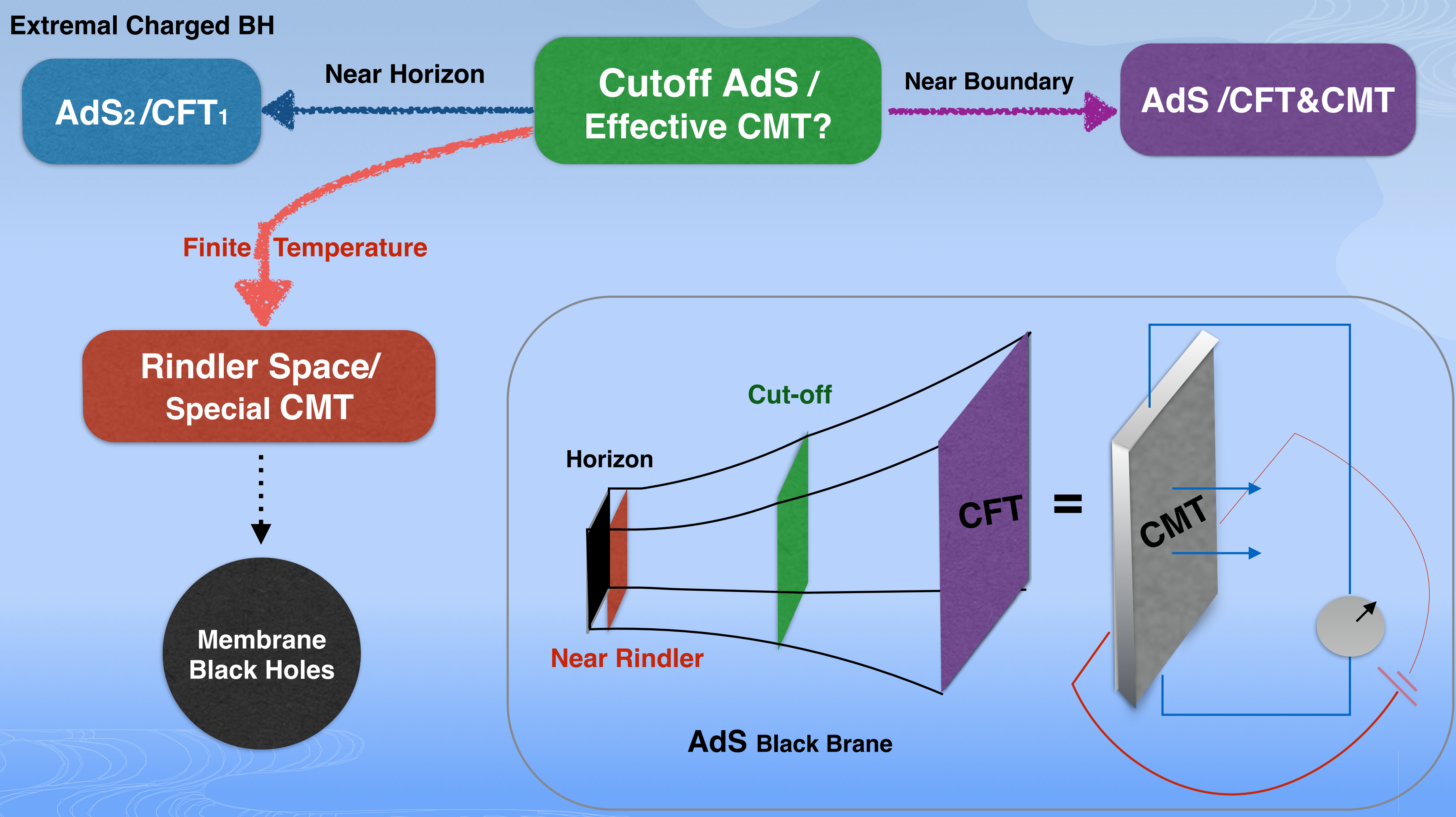


# Motivations



## Black Holes ⇔ Lower dimensional Fluid

- The horizon responds like a viscous fluid

Stress Tensor:

$$T_{ab} = 2(K\gamma_{ab} - K_{ab})$$

Hawking Temperature:

$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

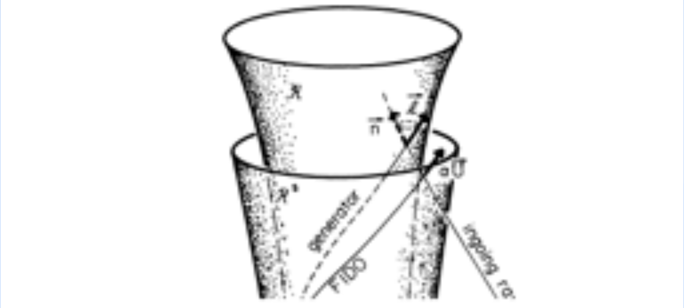


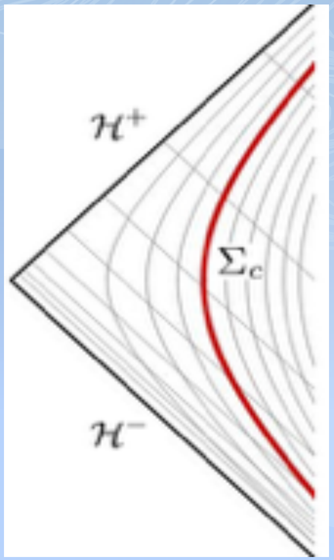

Figure: R. H. Price and K. S. Thorne, Phys. Rev. D 33, 915 (1986)

- Near horizon limit ➔ Rindler metric

$$ds^2_{p+2} = -r d\tau^2 + 2 d\tau dr + dx_i dx^i,$$

Figure（Pisin Chen)

# Rindler Hydrodynamics




- **Induced metric**

$$ds^2_{p+1} = \gamma_{ab}\mathrm{d}x_a\mathrm{d}x^b = -r_c\mathrm{d}\tau^2 + \mathrm{d}x_i\mathrm{d}x^i.$$

  - Dual Fluid: $T_{ab} = 2(K\gamma_{ab} - K_{ab}).$

- **Constraint equations**

$$2G_{\mu b}n^\mu|_{\Sigma_c} = 2(\partial^a K_{ab} - \partial_b K) = 0 \Longrightarrow \partial^a T_{ab} = 0,$$

$$2G_{\mu\nu}n^\mu n^\nu|_{\Sigma_c} = (K^2 - K_{ab}K^{ab}) = 0 \Longrightarrow T^2 - p\,T_{ab}T^{ab} = 0,$$


I. Bredberg, C. Keeler, V. Lysov, and A. Strominger, "From Navier-Stokes To Einstein," *JHEP* **1207** (2012) 146, arXiv:1101.2451 [hep-th].

# Derivative Expansion:

$$T_{ab} = T_{ab}^{(0)} + T_{ab}^{(1)} + T_{ab}^{(2)} + O(\partial^3),$$

$$T_{ab}^{(0)} = \mathrm{p} h_{ab},$$

$$T_{ab}^{(1)} = \zeta' (u^c \partial_c \ln \mathrm{p}) u_a u_b - 2\eta \mathcal{K}_{ab},$$

$$\begin{aligned} T_{ab}^{(2)} = \mathrm{p}^{-1} \Big\{ &\big[ d_1 \mathcal{K}_{ab} \mathcal{K}^{ab} + d_2 \Omega_{ab} \Omega^{ab} + d_3 (u^c \partial_c \ln \mathrm{p})^2 \\ &+ d_4 u^c \partial_c (u^d \partial_d \ln \mathrm{p}) + d_5 h^{cd} (\partial_c \ln \mathrm{p})(\partial_d \ln \mathrm{p}) \big] u_a u_b \\ &+ \big[ c_1 \mathcal{K}_{ac} \mathcal{K}^c{}_b + c_2 \mathcal{K}_{c(a} \Omega^c{}_{b)} + c_3 \Omega_{ac} \Omega^c{}_b + c_4 h_a^c h_b^d \partial_c \partial_d \ln \mathrm{p} \\ &+ c_5 \mathcal{K}_{ab} (u^c \partial_c \ln \mathrm{p}) + c_6 (h_a^c \partial_c \ln \mathrm{p})(h_b^d \partial_d \ln \mathrm{p}) \big] \Big\}. \end{aligned}$$

## Rindler Fluid

### Transport coefficients

$$\zeta' = 0, \qquad \eta = 1,$$

$$d_1 = -2, \quad d_2 = d_3 = d_4 = d_5 = 0,$$

$$c_1 = -2, \quad c_2 = c_3 = c_4 = c_5 = -c_6 = -4\,.$$

G. Compere, P. McFadden, K. Skenderis and M. Taylor, "The relativistic fluid dual to vacuum Einstein gravity," JHEP **1203**, 076 (2012) [arXiv:1201.2678 [hep-th]].

# Dual Metric:

- Shear

$$\mathcal{K}_{ab} = h_a^c h_b^d \partial_{(c} u_{d)},$$

- Vorticity

$$\Omega_{ab} = h_a^c h_b^d \partial_{[c} u_{d]},$$

- Constraint eqs.

$$\partial_a u^a = 2\mathfrak{p}^{-1} \mathcal{K}_{ab} \mathcal{K}^{ab}$$

$$a_a + h_a^b \partial_b \ln \mathfrak{p} = 2\mathfrak{p}^{-1} h_a^c \partial_b \mathcal{K}_c^b$$

- Einstein eqs.

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0,$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -2\mathfrak{p} u_a dx^a dr + g_{ab} dx^a dx^b,$$

where $g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)} + g_{ab}^{(2)}$,

$$g_{ab}^{(0)} = -\mathfrak{p}^2 (r - r_c) u_a u_b + \gamma_{ab},$$

$$g_{ab}^{(1)} = 2\mathfrak{p}(r - r_c)(u^c \partial_c \ln \mathfrak{p}\, u_a u_b + 2a_{(a} u_{b)}),$$

$$\begin{aligned} g_{ab}^{(2)} = {} & 2(r - r_c)[(\mathcal{K}_{cd}\mathcal{K}^{cd}) u_a u_b - 2u_{(a} h_{b)}^c \partial_d \mathcal{K}^d{}_c \\ & - \mathcal{K}_a{}^c \mathcal{K}_{cb} + 2\mathcal{K}_{c(a}\Omega^c{}_{b)} - 2h_a^c h_b^d u^e \partial_e \mathcal{K}_{cd}] \\ & + \mathfrak{p}^2 (r - r_c)^2 \left\{ \left( \frac{1}{2} \mathcal{K}_{cd} \mathcal{K}^{cd} + a_c a^c \right) u_a u_b \right. \\ & \left. + 2u_{(a} h_{b)}^c [\partial_d \mathcal{K}^d{}_c - (\mathcal{K}_{cd} + \Omega_{cd}) a^d] - \Omega_{ac}\Omega^c{}_b \right\} \\ & + \mathfrak{p}^4 (r - r_c)^3 \left( \frac{1}{2} \Omega_{cd} \Omega^{cd} \right) u_a u_b. \end{aligned}$$

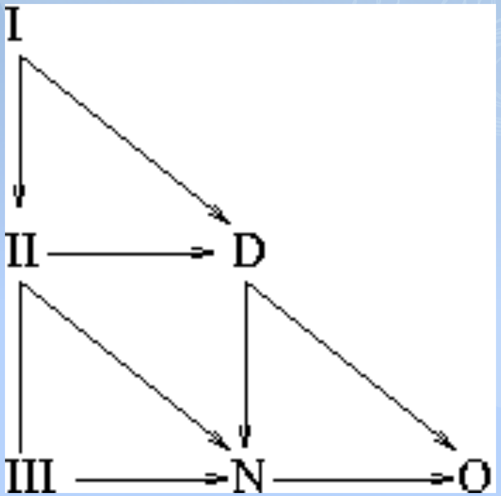
# More Simple & Universal Relation?

- A recursive relation between different orders?
  - Gravity ⇔ A special Fluid
  - Gravity ⇔ Riemannian Geometry
- Petrov type I condition!
  - No more gravitational field equations

V. Lysov and A. Strominger, "From Petrov-Einstein to Navier-Stokes," arXiv:1104.5502 [hep-

# Petrov type classification

- Algebraic Symmetry of Weyl tensors
  - By A. Z. Petrov (1954)
  - In 3+1 dimension




- Physical Interpretation
  - Type D: Isolated massive objects [Kerr vacuum]
  - Type III: longitudinal gravitational radiation.
  - Type N: transverse gravitational radiation [LIGO]
  - Type O: Conformally flat [electromagnetic field]

# Petrov type I condition

$$\mathbb{P}_{ij} = 0, \quad \mathbb{P}_{ij} \equiv C_{(\ell)i(\ell)j} \equiv \ell^\mu m_i^\nu \ell^\alpha m_j^\beta C_{\mu\nu\alpha\beta}.$$

- **The covariant formula:** $\mathbb{P}_{ij} = m_i{}^a m_j{}^b \mathbb{P}_{ab} = 0,$
  - After using Gauss-Codazzi equations
  - ➔ **A recursive relation**

$$\begin{aligned}2\mathbb{P}_{ab} = & h_a^c h_b^d \left[ \left( T_{mc} T_{nd} - T_{mn} T_{cd} \right) u^m u^n - T_{cm} T^m{}_d \right] \\ & + 4 h_a^c h_b^d \left[ - u^m \partial_m T_{cd} + u^m \partial_{(c} T_{d)m} \right] \\ & + p^{-2} \left[ T \left( T + p\, T_{mn} u^m u^n \right) + 4 p\, u^c \partial_c T \right] h_{ab}.\end{aligned}$$

  - **How about the Rindler Fluid?**

R. G. Cai, Q. Yang and Y. L. Zhang, "Petrov type I Spacetime and Dual Relativistic Fluids," Phys. Rev. D **90**, 041901 (2014) [arXiv:1401.7792 [hep-th]].

# We can recover the dual Fluid

- Decompose the stress tensor

$$\hat{T}_{ab} = \mathbb{e} u_a u_b + \Pi_{ab}, \quad \mathbb{e} \equiv \hat{T}_{ab} u^a u^b, \quad \Pi_{ab} \equiv h_a^c h_b^d \hat{T}_{cd}.$$

  - Hamiltonian constraint: $\mathbb{H} = 0,$

$$\mathbb{H} \equiv 2p\mathbb{e}\mathbb{p} + (p-1)\mathbb{e}^2 + p\Pi_{ab}\Pi^{ab} - p^2\mathbb{p}^2.$$

  - Petrov type I condition: $\mathbb{P}_{ab} = 0,$

$$2\mathbb{P}_{ab} \equiv -\mathbb{e}\Pi_{ab} - \Pi_{ac}\Pi^c{}_b - 4u^c\partial_c\Pi_{ab} - 4\Pi_{(a}{}^c D_{b)}^{\perp} u_c - 4\mathbb{e}\mathcal{K}_{ab} + \left[\mathbb{p}\left(\mathbb{e}+\mathbb{p}\right) + 4\mathbb{p}\, u^c\partial_c \ln \mathbb{p}\right] h_{ab}.$$

# Petrov type I condition → Rindler Fluid

- Give the 0th order $\mathbb{e}^{(0)} = 0,\ \Pi^{(0)}_{ab} = \mathbb{p} h_{ab}$
  - 1st order
    $$\mathbb{H}^{(1)} = 0 \Rightarrow \mathbb{e}^{(1)} = 0,$$
    $$\mathbb{P}^{(1)}_{ab} = 0 \Rightarrow \Pi^{(1)}_{ab} = -2\mathcal{K}_{ab},$$
  - 2nd order
    $$\mathbb{H}^{(2)} = 0 \Rightarrow \mathbb{e}^{(2)} = -2\mathbb{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab},$$
    $$\mathbb{P}^{(2)}_{ab} = 0 \Rightarrow \Pi^{(2)}_{ab} = \mathbb{p}^{-1}\big[-2\mathcal{K}_{ac}\mathcal{K}^{c}{}_{b} - 4\mathcal{K}_{c(a}\Omega^{c}{}_{b)} - 4\Omega_{ac}\Omega^{c}{}_{b} - 4h^{c}_{a}h^{d}_{b}\partial_c\partial_d \ln\mathbb{p} - 4\mathcal{K}_{ab}(u^c\partial_c\ln\mathbb{p}) + 4(h^{c}_{a}\partial_c\ln\mathbb{p})(h^{d}_{b}\partial_d\ln\mathbb{p})\big],$$

**The Stress Tensor:**

$$\hat{T}_{ab} = \mathbb{e}^{(2)}u_a u_b + \mathbb{p} h_{ab} + \Pi^{(1)}_{ab} + \Pi^{(2)}_{ab}.$$

# Compare with the general stress tensor

$$T^{(0)}_{ab} = 0 + \mathbb{p} h_{ab},$$

$$T^{(1)}_{ab} = (\zeta' D \ln \mathbb{p}) u_a u_b - 2\eta \mathcal{K}_{ab},$$

$$\begin{aligned} T^{(2)}_{ab} = \mathbb{p}^{-1}\Big\{ \Big[ & d_1 \mathcal{K}_{ab}\mathcal{K}^{ab} + d_2 \Omega_{ab}\Omega^{ab} + d_3 (D \ln \mathbb{p})^2 \\ & + d_4 DD \ln \mathbb{p} + d_5 (D_\perp \ln \mathbb{p})^2 \Big] u_a u_b + \big( c_1 \mathcal{K}_{ac}\mathcal{K}^c{}_b \\ & + c_2 \mathcal{K}_{c(a}\Omega^c{}_{b)} + c_3 \Omega_{ac}\Omega^c{}_b + c_4 h^c_a h^d_b \partial_c \partial_d \ln \mathbb{p} \\ & + c_5 \mathcal{K}_{ab} D \ln \mathbb{p} + c_6 D^\perp_a \ln \mathbb{p}\, D^\perp_b \ln \mathbb{p} \big) \Big\}. \end{aligned}$$

**Exactly the same coefficients from fluid/gravity!**

$$\zeta' = 0, \qquad \eta = 1,$$

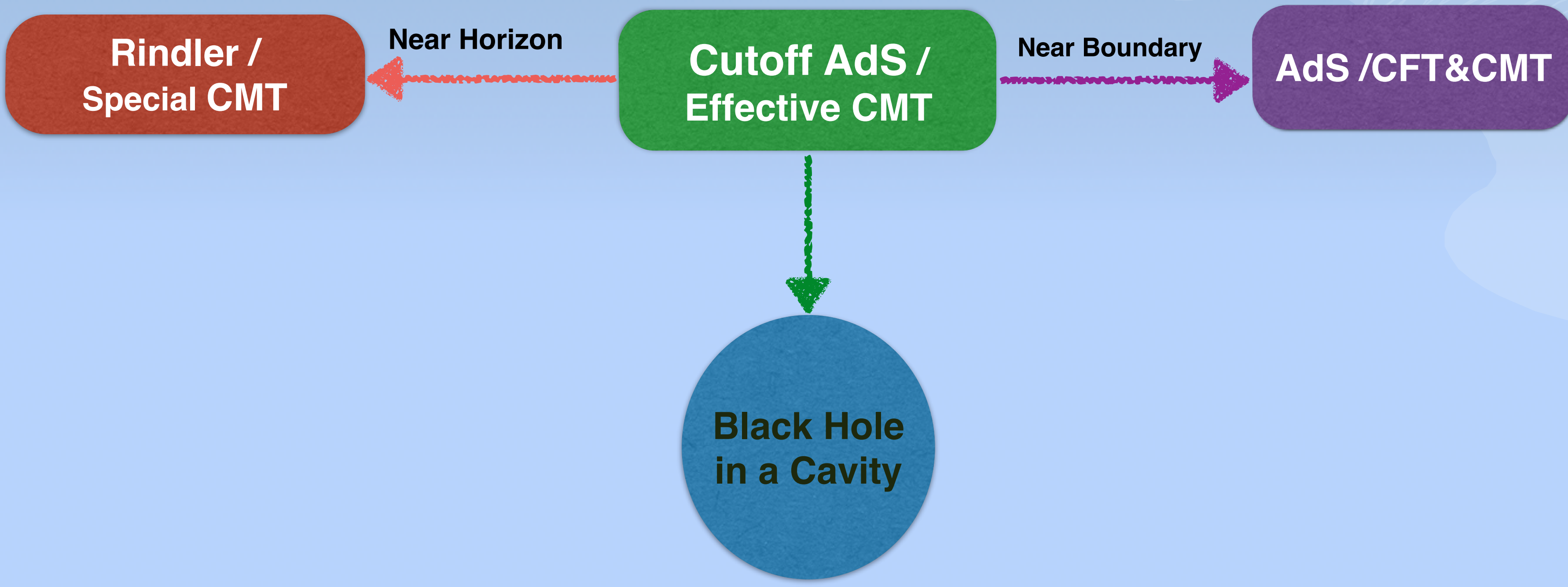
$$d_1 = -2, \quad d_2 = d_3 = d_4 = d_5 = 0,$$

$$c_1 = -2, \quad c_2 = c_3 = c_4 = c_5 = -c_6 = -4.$$

# Conclusions

- A recursive relation of Rindler fluid
  - AdS/Fluid & Charged Fluid
  - Higher curvature corrections
- Special properties near the Horizon regions
  - Equivalent choice of the regularity condition?
  - A criterion of black holes observation?

# Outlook




Towards Holography for Black Hole in a Box

- **Thanks for your attention!**

# Generalization to higher dimensions

- A. Coley, et al.(2004)

$$\text{Type I}: \quad C^{[2]}_{\alpha\beta\gamma\delta} = 0,$$
$$\text{Type II}: \quad C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = 0,$$
$$\text{Type D}: \quad C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[-1]}_{\alpha\beta\gamma\delta} = C^{[-2]}_{\alpha\beta\gamma\delta} = 0,$$
$$\text{Type III}: \quad C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[0]}_{\alpha\beta\gamma\delta} = 0,$$
$$\text{Type N}: \quad C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[0]}_{\alpha\beta\gamma\delta} = C^{[-1]}_{\alpha\beta\gamma\delta} = 0,$$
$$\text{Type O}: \quad C^{[2]}_{\alpha\beta\gamma\delta} = C^{[1]}_{\alpha\beta\gamma\delta} = C^{[0]}_{\alpha\beta\gamma\delta} = C^{[-1]}_{\alpha\beta\gamma\delta} = C^{[-2]}_{\alpha\beta\gamma\delta} = 0.$$

-

$$C_{\alpha\beta\gamma\delta} = C^{[2]}_{\alpha\beta\gamma\delta} + C^{[1]}_{\alpha\beta\gamma\delta} + C^{[0]}_{\alpha\beta\gamma\delta} + C^{[-1]}_{\alpha\beta\gamma\delta} + C^{[-2]}_{\alpha\beta\gamma\delta},$$

# Newman–Penrose-like formalism

$$\boldsymbol{\ell}^2 = \boldsymbol{k}^2 = 0, \quad (\boldsymbol{k}, \boldsymbol{\ell}) = 1, \quad (\boldsymbol{m}_i, \boldsymbol{k}) = (\boldsymbol{m}_i, \boldsymbol{\ell}) = 0, \quad (\boldsymbol{m}_i, \boldsymbol{m}_j) = \delta_{ij},$$

$$g_{\mu\nu} = 2\boldsymbol{\ell}_{(\mu}\boldsymbol{k}_{\nu)} + \delta_{ij}\boldsymbol{m}^i{}_\mu \boldsymbol{m}^j{}_\nu, \qquad g^{\mu\nu} = 2\boldsymbol{\ell}^{(\mu}\boldsymbol{k}^{\nu)} + \delta^{ij}\boldsymbol{m}_i{}^\mu \boldsymbol{m}_j{}^\nu.$$

$$\begin{aligned}
C^{[2]}_{\alpha\beta\gamma\delta} =& 4C_{(\ell)i(\ell)j}\boldsymbol{k}_{\{\alpha}\boldsymbol{m}^i{}_\beta \boldsymbol{k}_\gamma \boldsymbol{m}^j{}_{\delta\}},\\
C^{[1]}_{\alpha\beta\gamma\delta} =& 8C_{(\ell)(k)(\ell)i}\boldsymbol{k}_{\{\alpha}\boldsymbol{\ell}_\beta \boldsymbol{k}_\gamma \boldsymbol{m}^i{}_{\delta\}} + 4C_{(\ell)ijk}\boldsymbol{k}_{\{\alpha}\boldsymbol{m}^i{}_\beta \boldsymbol{m}^j{}_\gamma \boldsymbol{m}^k{}_{\delta\}},\\
C^{[0]}_{\alpha\beta\gamma\delta} =& 4C_{(\ell)(k)(\ell)(k)}\boldsymbol{k}_{\{\alpha}\boldsymbol{\ell}_\beta \boldsymbol{k}_\gamma \boldsymbol{\ell}_{\delta\}} + 4C_{(\ell)(k)ij}\boldsymbol{k}_{\{\alpha}\boldsymbol{\ell}_\beta \boldsymbol{m}^i{}_\gamma \boldsymbol{m}^j{}_{\delta\}}\\
&+8C_{(\ell)i(k)j}\boldsymbol{k}_{\{\alpha}\boldsymbol{m}^i{}_\beta \boldsymbol{\ell}_\gamma \boldsymbol{m}^j{}_{\delta\}} + C_{ijkl}\boldsymbol{m}^i_{\{\alpha}\boldsymbol{m}^j{}_\beta \boldsymbol{m}^k{}_\gamma \boldsymbol{m}^l{}_{\delta\}},\\
C^{[-1]}_{\alpha\beta\gamma\delta} =& 8C_{(k)(\ell)(k)i}\boldsymbol{\ell}_{\{\alpha}\boldsymbol{k}_\beta \boldsymbol{\ell}_\gamma \boldsymbol{m}^i{}_{\delta\}} + 4C_{(k)ijk}\boldsymbol{\ell}_{\{\alpha}\boldsymbol{m}^i{}_\beta \boldsymbol{m}^j{}_\gamma \boldsymbol{m}^k{}_{\delta\}},\\
C^{[-2]}_{\alpha\beta\gamma\delta} =& 4C_{(k)i(k)j}\boldsymbol{\ell}_{\{\alpha}\boldsymbol{m}^i{}_\beta \boldsymbol{\ell}_\gamma \boldsymbol{m}^j{}_{\delta\}},
\end{aligned}$$

# Recursive relation for dual fluid?

- Constitute relation of stress tensor

$$T^{ab}(x) = \mathcal{E}(x)u^a(x)u^b(x) + \mathcal{P}(x)P^{ab}(x) + \Pi^{ab}_{\langle\partial\rangle}(x)$$

- Conservation equation: $\nabla_a T^{ab} = 0.$
- Energy density ⬄ Pressure: State equation

First dissipative order: $\Pi^{ab}_{(1)} = -2\,\eta\,\sigma^{ab} - \zeta\,\theta\,P^{ab}$

$$\sigma^{ab} = \nabla^{\langle a}\,u^{b\rangle} \equiv P^{ac}\,P^{bd}\left(\nabla_{(c}\,u_{d)} - \frac{1}{p}\,\theta\,P_{cd}\right), \qquad \theta \equiv \nabla_c\,u^c,$$