# Observables in Gauge/Gravity Duality

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Target:

To study universal behavior of observables among different theories.

# Outline

# Introduction

- To examine certain properties of any theory we need to interact with it and compute expectation values. We do the same in gauge/gravity correspondence.
- One step further is to extract information for a large class of theories using common properties of the observables!
- In deconfined phase the dual theories have common characteristics, strongly coupled; no susy; no confinement. And differences: E.g. Number flavors, number of degrees of freedom etc. These may not, or weakly affect the observable in certain comparison schemes.

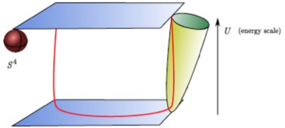
# Gauge/Gravity Duality: Several Examples

- ✓ Less Supersymmetry. Examples:
  a)Sasaki-Einstein solutions $AdS_5 \times Y^{p,q}$, $N = 1$ supersymmetry.
  b)TsT deformation on every background with global $U(1) \times U(1)$ symmetry. eg. $\beta$ deformation: $AdS_5 \times \tilde{S}^5$.
- ✓ Additional branes to the original theory.
  E.g. Inclusion of Anisotropy with anisotropically distributed heavy branes

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $u$ | $S^5$ |
|---|---|---|---|---|---|---|
| D3 | x | x | x | x | | |
| D7 | x | x | x | | | x |

Very interesting theories, several new features!

✓ Broken conformal symmetry, confinement.
Example: $D4$ Witten model.

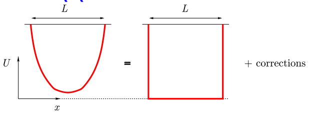



*(Fig: 0708.1502)*

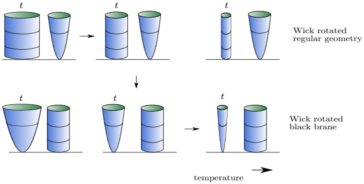
$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu} + f(u)dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4\right), \quad f(u) = 1 - \left(\frac{u_k}{u}\right)^3$$

The potential of the static heavy meson is linear:

$$V_{Q\bar{Q}} = \sigma L + \mathcal{O}(e^{-L}) \quad .$$



✓ Finite temperature. Presence of Black hole.




Example: An anisotropic black hole: *(Mateos, Trancanelli, 2011)*

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\, dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H} d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\, d\Omega^2_{S^5}\,.$$

The anisotropic parameter is $\alpha$ appears in the axion $(\chi = \alpha x_3)$. In high temperatures, $T \gg \alpha$:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 F_1(u, u_h)\,,$$

$$\mathcal{B}(u) = 1 - \alpha^2 B_1(u, u_h), \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

# General Theory for Observables

- Assume the existence of the gravity dual of a theory.
- For most of the observables we can work in full generality

$$ds^2 = g_{00}(u)dx_0^2 + \sum g_{ii}(u)dx_i^2 + g_{pp}(u)dx_p^2 + g_{uu}(u)du^2 + \text{internal space}$$

  The $x_p$ is a chosen space direction.
- The background may have RR fluxes as well a non-trivial dilaton.
- String-related observables do no couple to them, while brane observables may have certain simplifications.

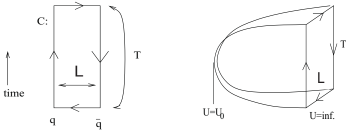
# Static Potential

Warm-up observable:

- The string world-sheet $(\tau, \sigma)$ of the following form.

## String Configuration

$$x_0 = \tau, \quad x_p = \sigma, \quad u = u(\sigma)\ , \quad u(0) = u(L) = u_{Boundary}$$




The solution to Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-g_{00}(g_{pp} + g_{uu} u'^2)}$$

is a catenary shape w-s with $u_0$ being the turning point.

In general the length of the two endpoints of the string on the boundary is given by

$$L_p = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu} c_0^2}{(g_{00} g_{pp} + c_0^2) g_{pp}}}$$

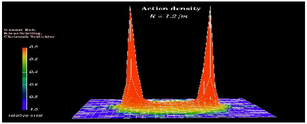
$c_0 \sim \sqrt{g_{00} g_{11}}_{(u_0)}$. $L_p$ should be inverted as $u_0(L_p)$ to find the normalized energy of the string is

$$2\pi\alpha' V_p(L_p) = c_0 L_p + 2\left[\int_{u_0}^{\infty} du \sqrt{-g_{uu} g_{00}} \left(\sqrt{1 + \frac{c_0^2}{g_{pp} g_{00}}} - 1\right) - \int_{u_h}^{u_0} du \sqrt{-g_{00} g_{uu}}\right]$$

*(Sonnenschein 2000, ... )*

# Width of the Chromoelectric flux tube

The chromoelectric field energy density between the $Q\bar{Q}$ is confined.




*(Fig:Bali, Schilling, Schlichter, 1995)*

To measure it we use a small probe Wilson loop $P(c)$, transverse to the WL at distance $\Delta x_3$ that corresponds to the heavy quark pair $W(C)$

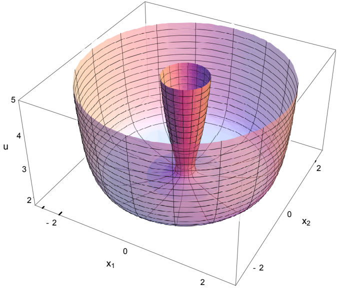
$$\mathcal{S}(x) = \frac{\langle W(C)P(c)\rangle - \langle W(C)\rangle\langle P(c)\rangle}{\langle W(C)\rangle} \, .$$

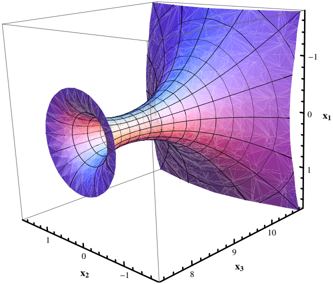
*(Lüsher, Munster, Weisz 1980)*

The mean square width of the flux tube is then defined as

$$w^2 = \frac{\int d(\Delta x_3)\Delta x_3^2 \; \mathcal{S}}{\int d(\Delta x_3) \; \mathcal{S}} \, .$$

Holographically we compute the connected minimal surface between two circles with radii $R \gg r_0$.




The system of equations in static gauge: $\theta = \tau,\ x_3 = \sigma; (x_1, x_2) \to (r, \theta)$

$$r'' - hr = 0\ ,$$

$$2u'' + u'^2 \partial_u (\ln f) - r^2 \frac{\partial_u h}{f} = 0\ ,$$

$$r'^2 + f u'^2 = hr^2 - 1\ .$$

with, $r(\sigma)$ the radii of the circles, $u(\sigma)$ the holographic coordinate and

$$h(u) := \frac{g_{11}^2}{c^2}, \qquad f(u) := \frac{g_{uu}}{g_{11}}\ .$$

For any confining background using its properties

$$S = \frac{g_{11}(u_k)}{2}\left(\frac{\Delta x_3}{\sqrt{h}} + R^2 - r_0^2 + \frac{1}{2}\left(1 - \epsilon^{-2\sqrt{h}\Delta x_3}\right)\right) \simeq \sigma\left(\frac{\Delta x_3^2}{\log\frac{R}{r_0}} + R^2 - r_0^2\right),$$

Resulting the logarithmic broadening

$$w^2 \simeq \frac{1}{2\pi\sigma}\log\frac{R}{r_0} \;,$$

Universal feature for any confining holographic theory! (*D.G.,Irges 2015*)

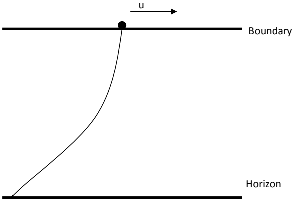
✓ Logarithmic Broadening in lattice Computations. e.g. (*Gliozzi, Pepe, Wiese, 2010;...*)

# Deconfined phase

The dynamics and the interactions of the heavy quark can be described by a diffusion treatment. The thermal momentum of the quark is $p_{th}^2 \simeq 3 m_Q T \gg T^2$. The momentum transfer of the medium is $Q^2 \simeq T^2$. Brownian motion of the heavy quark in a light particle fluid.

$$\frac{dp}{dt} = F_{drag} + F(t) \ .$$

The drag force of a single quark moving in the plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. (*Gubser, 2006*)



The drag force of a quark moving along the $p$ direction, for any background is given by the momentum flowing from the boundary to the bulk

$$F_{drag,p} = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0}$$

where $u_0$ is given by

$$\left(g_{uu}(g_{00} + g_{pp}v^2)\right)\,|_{u=u_0} = 0\ .$$

At $u = u_0$ there is horizon of the induced worldsheet metric. The 'effective world-sheet temperature' is

$$T_{ws}^2 = \left|\frac{1}{16\pi^2}\frac{1}{g_{00}g_{uu}}(g_{00}\,g_{pp})'\left(\frac{g_{00}}{g_{pp}}\right)'\right|\Bigg|_{u=u_0}\ .$$

In near horizon Dp black brane geometries $T_{ws} < T$. (*Nakamura, Ooguri 2013*)

# Momentum Broadening

The $F(t)$ is the factor that causes the momentum broadening, which leads to

$$\frac{\left\langle p_{L,T}^2 \right\rangle}{\mathcal{T}} = 2\kappa_{L,T}$$

$\kappa =$ Mean Squared Momentum Transfer per Time.

• The index $L$ refers to the direction along the motion of quark, the index $T$ is the direction transverse to the velocity of quark.

• In strong coupling limit for a quark moving along $p$ direction, these fluctuations are introduced to the Wilson line

$$t = \tau, \quad u = \sigma, \quad x_p = v\ t + \xi(\sigma) + \delta x_p(\tau, \sigma), \quad x_k = \delta x_k(\tau, \sigma)\ .$$

$\delta x_p(\tau, \sigma)$ : Longitudinal fluctuation.
$\delta x_k(\tau, \sigma)$ : Transverse fluctuation .

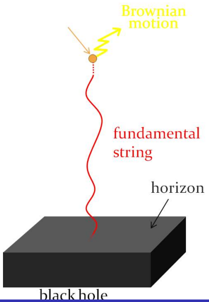
The Nambu-Goto action in fluctuations around the solution to quadratic order becomes

$$S_2 = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\, \frac{\tilde{G}^{\alpha\beta}}{2}\left[N(u)\,\partial_\alpha\delta x_p\,\partial_\beta\delta x_p + \sum_{i=2,3} g_{ii}\partial_\alpha\delta x_i\,\partial_\beta\delta x_i\right]$$

where

$$\tilde{G}^{\alpha\beta} = \sqrt{-\tilde{g}}\,\tilde{g}^{\alpha\beta}, \qquad N(u) = \frac{g_{00}\,g_{pp} + C^2}{g_{00} + g_{pp}\,v^2},$$

$\tilde{g}^{\alpha\beta}$ is the world-sheet metric, expressed in metric elements of the background.




The Langevin coefficients for a quark moving along the $p$ direction and the transverse direction is taken to be $k$:

$$\kappa_T = \frac{1}{\pi\alpha'}\, g_{kk}\Big|_{u=u_0} T_{ws}\,,$$

$$\kappa_L = \frac{16\,\pi}{\alpha'}\,\frac{|g_{00}|\,g_{uu}}{g_{pp}\left(\frac{g_{00}}{g_{pp}}\right)'^2}\Bigg|_{u=u_0} T_{ws}^3\,.$$

Their ratio can be simplified to

$$\frac{\kappa_L}{\kappa_T} = \frac{1}{g_{pp} g_{kk}} \frac{(g_{00} g_{pp})'}{(g_{00}/g_{pp})'}\bigg|_{u=u_0}$$

Example: $p = 3$ and $k = 1$: Quark moves along the direction $x_3$, and the transverse direction to motion is $x_1$.

•For any isotropic theory $g_{pp} = g_{kk}$ and $g_{00} = g_{00,bh}\ g_{kk}\ f$, we prove $\kappa_L > \kappa_T$ .

• This is a Universal Inequality independent of the background used!

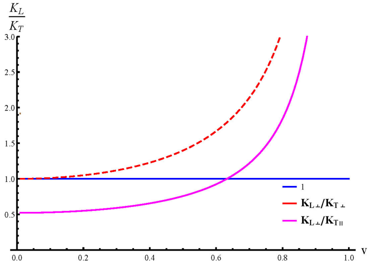
*(D.G, Soltanpanahi, 2013a; Gursoy, Kiritsis, Mazzanti, Nitti, 2010)*

• The only possibility to have violation of the inequality is in the anisotropic theories!

• In fact the motion of the quark in the axion deformed anisotropic theory violates the inequality! *(D.G, Soltanpanahi, 2013b)*

Consider the space dependent axion anisotropic background.

- $\kappa_L < \kappa_T$ when the motion of the quark is along the transverse to the anisotropic direction.



# Conclusions

Working with a large class of theories we obtain universal behaviors.

- Logarithmic flux tube broadening, in confining theories.
- The Universal Langevin coefficients inequality $\kappa_L > \kappa_T$ proved to hold for isotropic backgrounds is violated for the anisotropic theories!

  Similar treatment:
- Non-Integrability for large class of backgrounds. *(D.G, Sfetsos 2014)*.
- $k$-strings = fundamental strings with effective string tension for large class of theories. *(D.G, 2015)*.

# Thank you