Entanglement With Centers

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$$H \sim \bigoplus_k \left(H_V^k \otimes H_{\bar{V}}^k \right)$$

$$
\begin{aligned}
-\mathrm{Tr}\big(\rho_A \ln \rho_A\big) &= -\sum_k \mathrm{Tr}\big(p_k \rho_{A_k} \ln(p_k \rho_{A_k})\big) \\
&= -\sum_k p_k \ln p_k - \sum_k \mathrm{Tr}\big(p_k \rho_{A_k} \ln \rho_{A_k}\big).
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-\sum_\phi \left(f(\phi)\Delta\right) \ln(f(\phi)\Delta) \longrightarrow -\ln(\Delta) - \int d\phi \, f(\phi) \ln f(\phi).
$$

Lagrangian Method

The non-trivial center is equivalent to mentioning that we do not have quantum fluctuation in entangling surface. We can consider classical solution with boundary and bulk, and quantum fluctuation with bulk to compute the entanglement entropy.

The Result of Massless p-form Theory

The massless p-form theory in $(2p+2)$-dimensions has electric-magnetic duality. From the Hamitonian formulation, we can also find equivalent entanglement entropy in different centers from electric-magnetic-like duality.

Strong Coupling Expansion

$$H_{\text{LYMF}} = \frac{g^2}{2} \sum_{l} E_l^a E_l^a - \frac{1}{g^2} \sum_{\Box} \left(\text{Tr}\, U_{\Box} + \text{Tr}\, U_{\Box}^{\dagger} \right) .$$

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$$\psi'(U_{l_1},\cdots,U_{l_{\partial A}},U_{l_{\partial \bar{A}}},\cdots,U_{l_k}) \equiv \psi(U_{l_1},\cdots,U_{l_{\partial A}}\cdot U_{l_{\partial \bar{A}}},\cdots,U_{l_k}) ,$$

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$$S_{\mathrm{EE}} = n_A (D-2) \lambda^2 (-\ln \lambda^2 + 1 + 2 \ln N) + O(\lambda^3),$$

where n_A is the number of boundary links in region A and $\lambda \equiv \frac{2N}{g^4(N^2-1)}$.

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- The result of the massless p-form theory gives us a global symmetry structure in centers.
- The issue of the entanglement entropy in strong coupling region.