Universal Turbulence on Branes in Holography

[arXiv:1504.07836]

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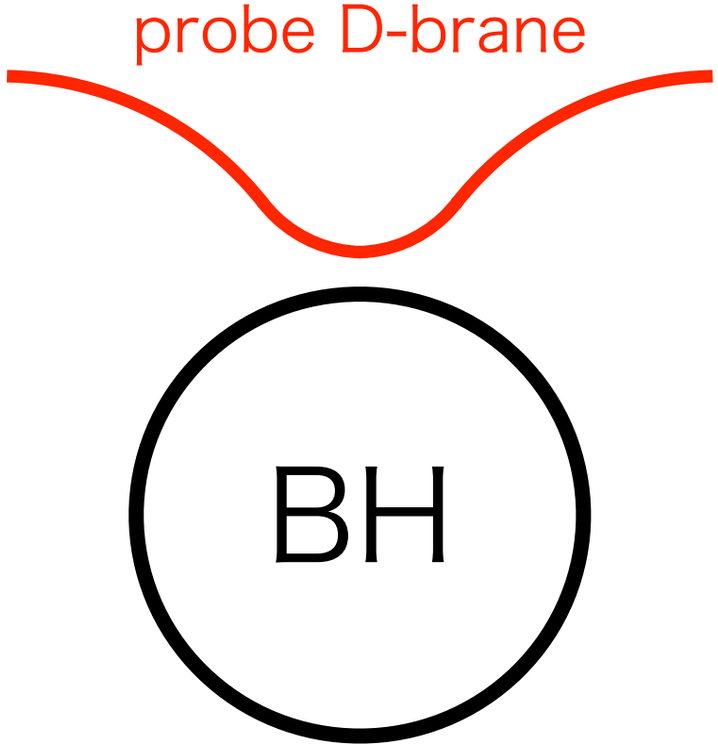
Phase transition of D-brane systems in holography

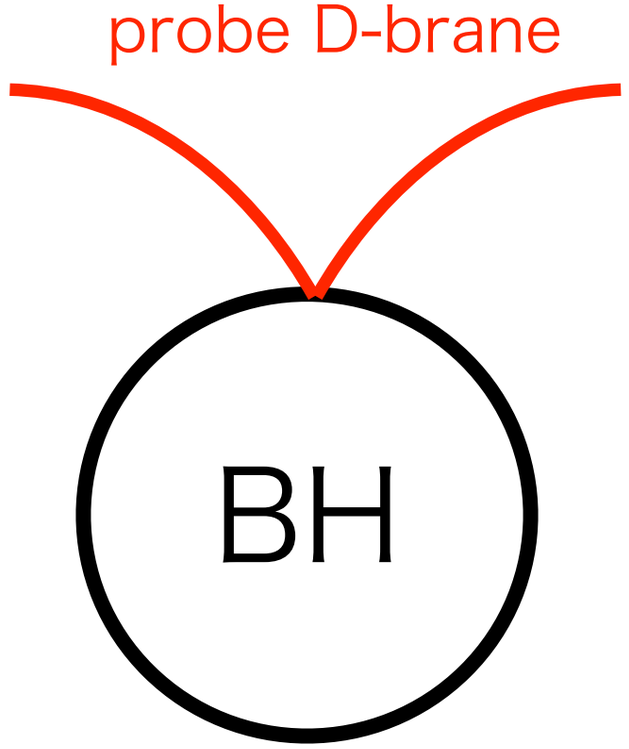
[Frolov, 2006] [Mateos, Myers, Thomson, 2006]

[Erdmenger, Meyer, Shock, 2007]

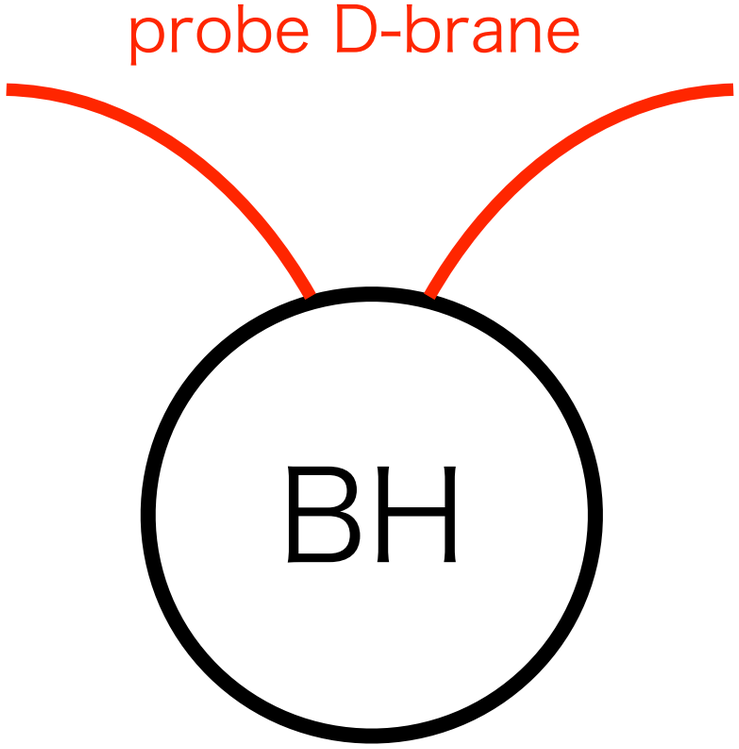
[Albash, Filev, Johnson, Kundu 2007]



Minkowski embedding



Critical embedding



Black hole embedding

fluctuation of probe D-brane $\approx$ meson field

At the critical embedding,
a power law appears
in meson energy spectrum of D3/D7.

(Turbulence meson condensation)

[Hashimoto, Kinoshita, Murata, Oka, 2014]

$$\varepsilon_n \propto \omega_n^{-5}$$

meson's energy　　meson's mass

Questions

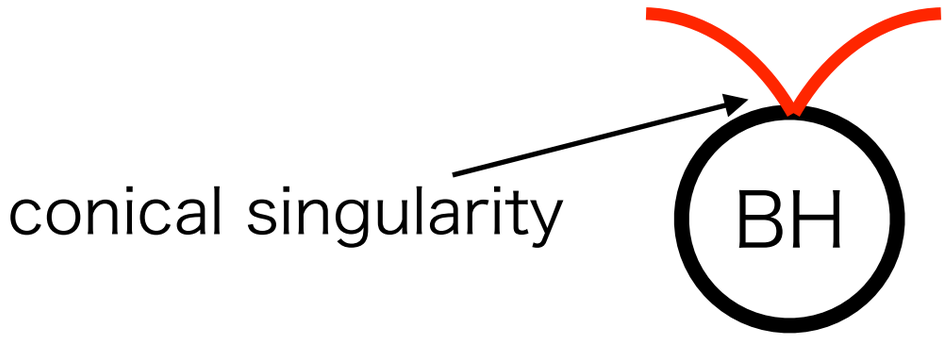
1. Does the power law appear
   in other D-brane systems?

2. Does the value of the power
   change in other D-brane systems?

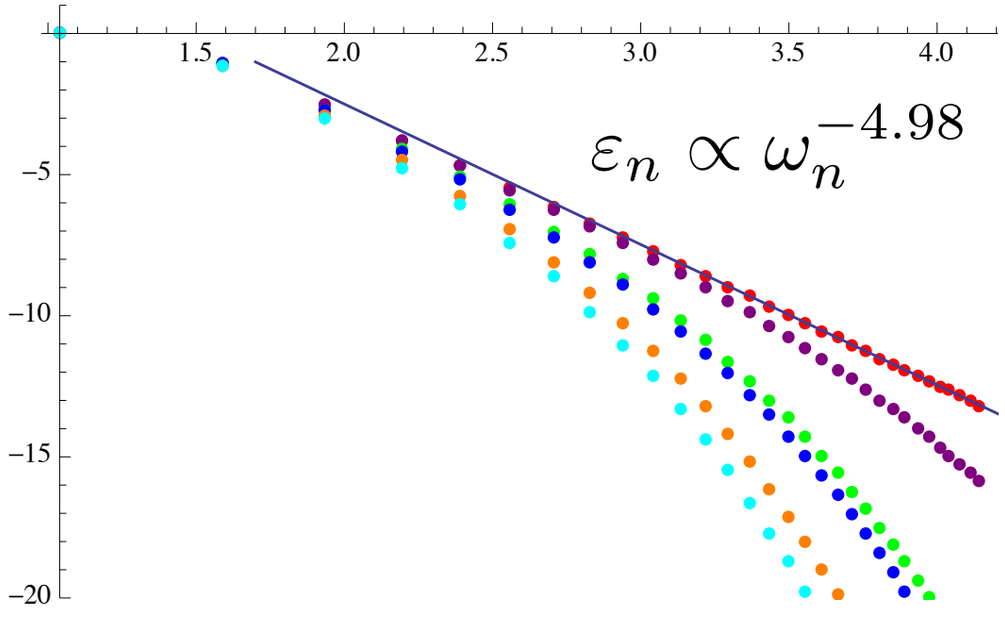
We analyze EOM of DBI action on the gravitational background.

Our result

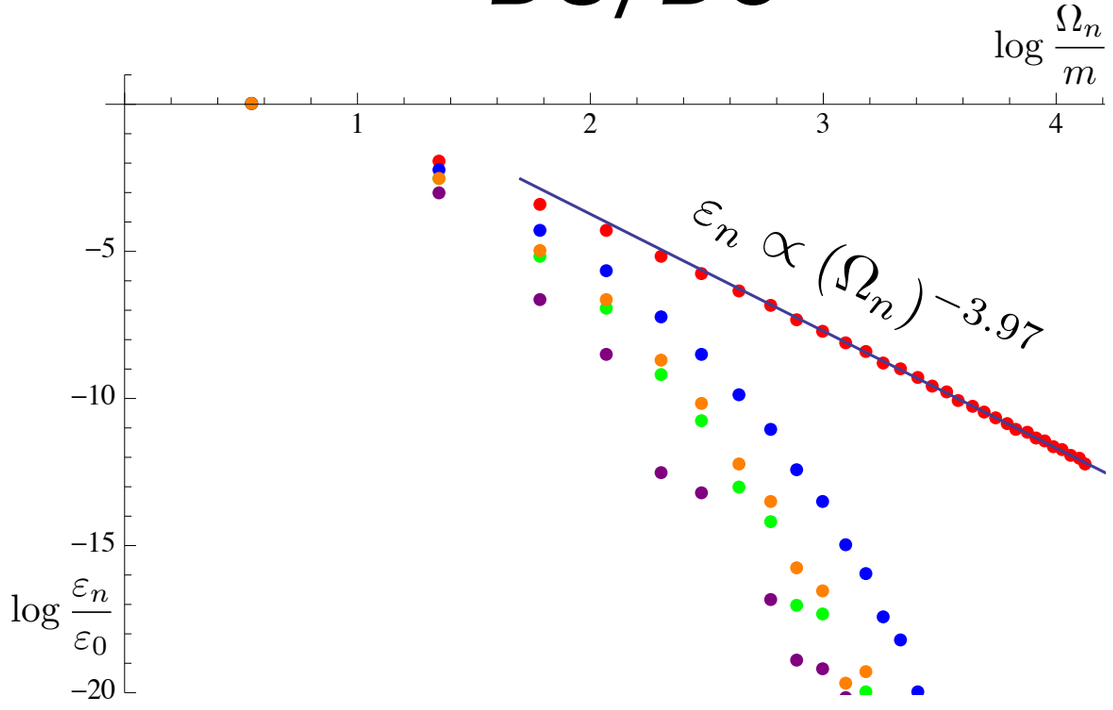
1. We find that the power law appears
   in D3/D7 and D3/D5 at the critical embedding.
2. The power depends on the cone dimensions
   of the probe D-branes.



D3/D7



D3/D5



1.Introduction

2.What is turbulent
meson condensation？

3.D-brane systems
and results

What is turbulence？

- random flow
- vortex
- nonlinear equation

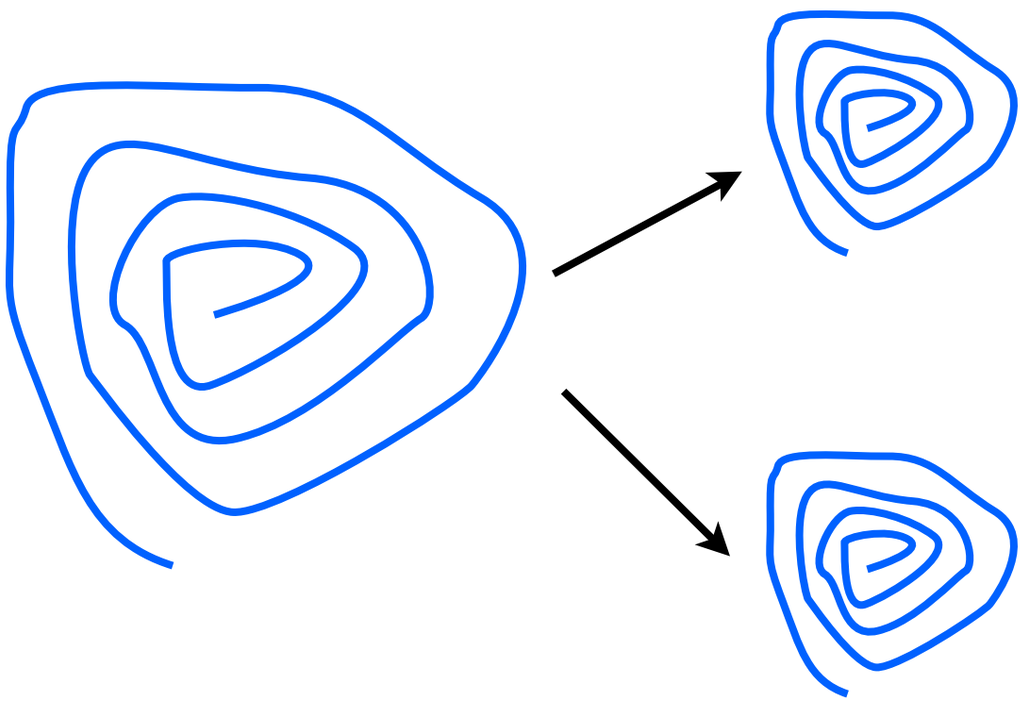
https://www.oist.jp/photo/eddies-turbulent-pool-sketch-15th-century-leonardo-da-vinci

But, there is no clear definition of turbulence.

Definition of turbulence in this talk

- energy flow from low to high wavenumber
- power law

From large vortex
to small vortex



Kolmogorov's power law
for turbulence

$$E(k) = C\varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

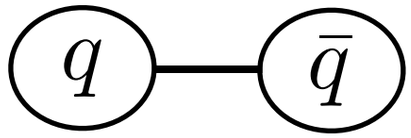
energy spectrum of vortex

wave number

Our aim

Compute meson energy spectrum
by using holography
and find power law at the critical embedding.
(Turbulence meson condensation)

vortex



meson

wavenumber
of vortex

eigenmass
of meson

1.Introduction

2.What is turbulent
meson condensation？

3.D-brane systems
and results

D3/D7 brane system

background
D3-branes

+

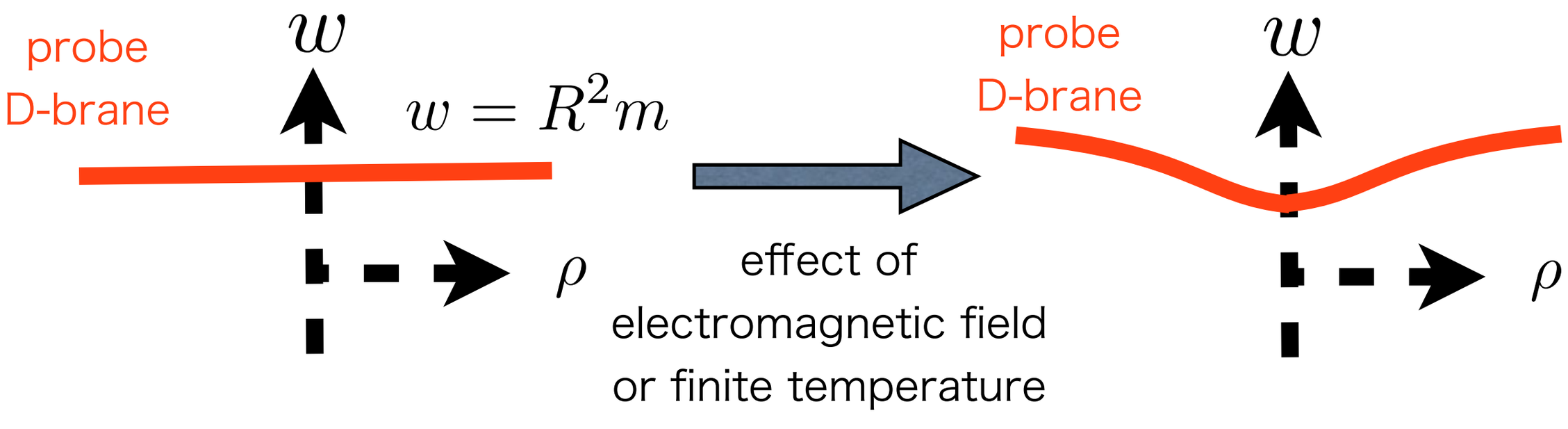
probe
D7-brane

$AdS_5 \times S^5$ metric

DBI action

$$ds^2 = \frac{\rho^2 + w^2 + \bar{w}^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2 + w^2 + \bar{w}^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\bar{w}^2]$$

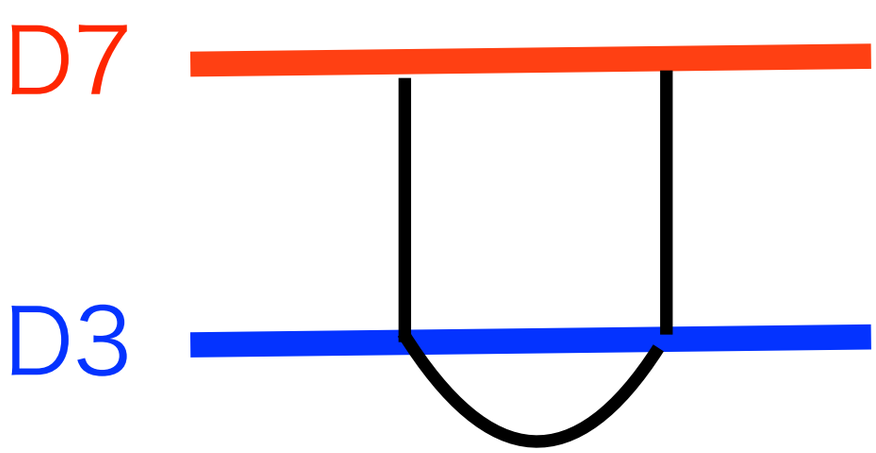
$$S = \frac{-1}{(2\pi)^6 g_{\mathrm{YM}}^2 l_s^8}\int d^8\xi\sqrt{-\det(g_{ab}[w] + 2\pi l_s^2 F_{ab})}$$

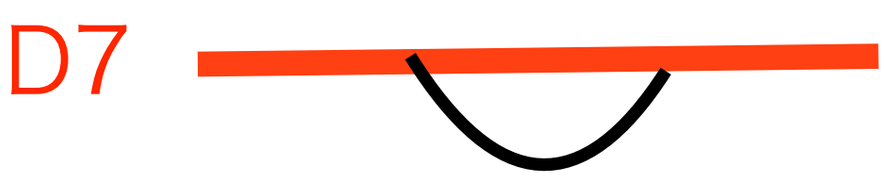


fluctuation of probe D-brane $\approx$ meson field

D3/D7

D7 on $AdS_5 \times S^5$





bound state of open strings $\approx$ meson

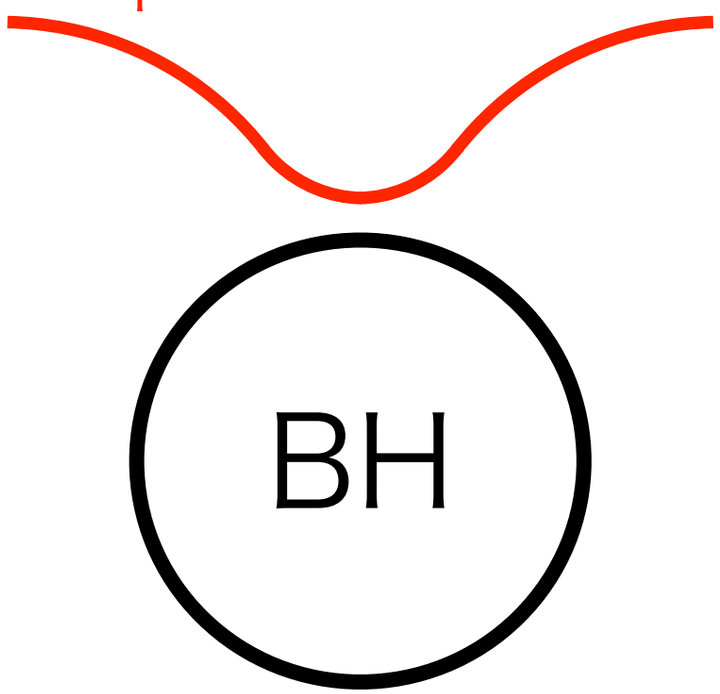
Three solutions

$AdS_5 \times S^5$ metric

$$ds^2 = \frac{\rho^2 + w^2 + \bar{w}^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2 + w^2 + \bar{w}^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\bar{w}^2]$$

DBI action

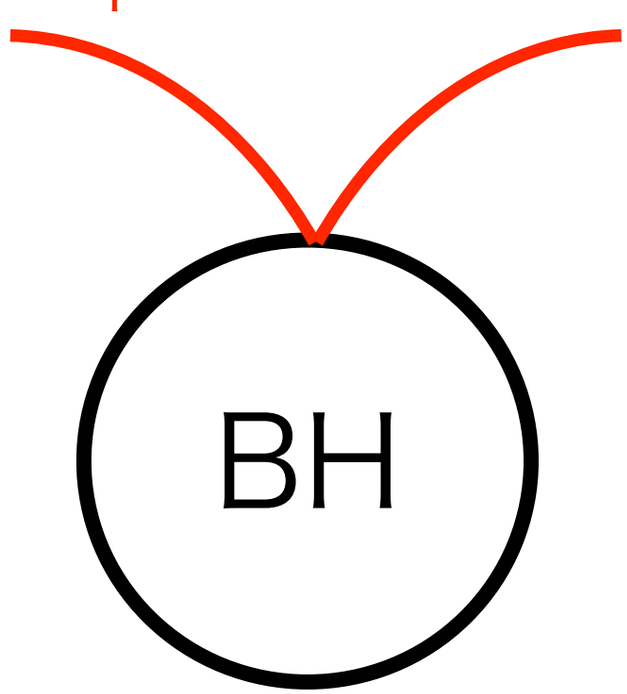
$$S = \frac{-1}{(2\pi)^6 g_{\mathrm{YM}}^2 l_s^8}\int d^8\xi\sqrt{-\det(g_{ab}[w] + 2\pi l_s^2 F_{ab})}$$



Minkowski embedding

$\sqrt{-\det(g_{ab}[w] + 2\pi l_s^2 F_{ab})} > 0$

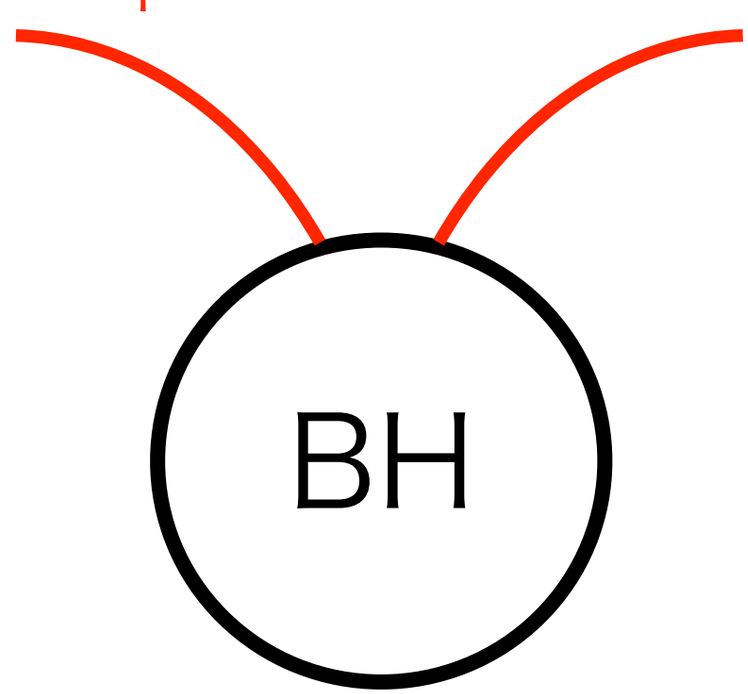
at $\rho = 0$



Critical embedding

$\sqrt{-\det(g_{ab}[w] + 2\pi l_s^2 F_{ab})} = 0$

at $\rho = 0$



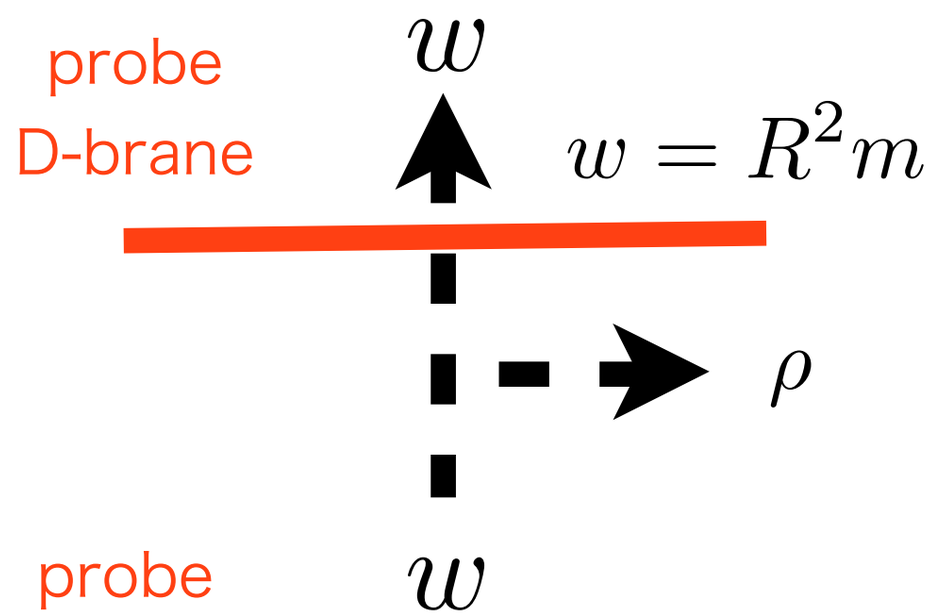
Black hole embedding

$\sqrt{-\det(g_{ab}[w] + 2\pi l_s^2 F_{ab})} = 0$

at $\rho > 0$

Analysis method

(1) Calculate meson's mass ω_n
and eigenfunction $e_n(\rho)$
without external fields.



(2) Solve EOM of $w(\rho)$ numerically
with nonzero external fields.

(3) Expand the solution by the
eigenfunction and calculate
meson's energy ε_n.

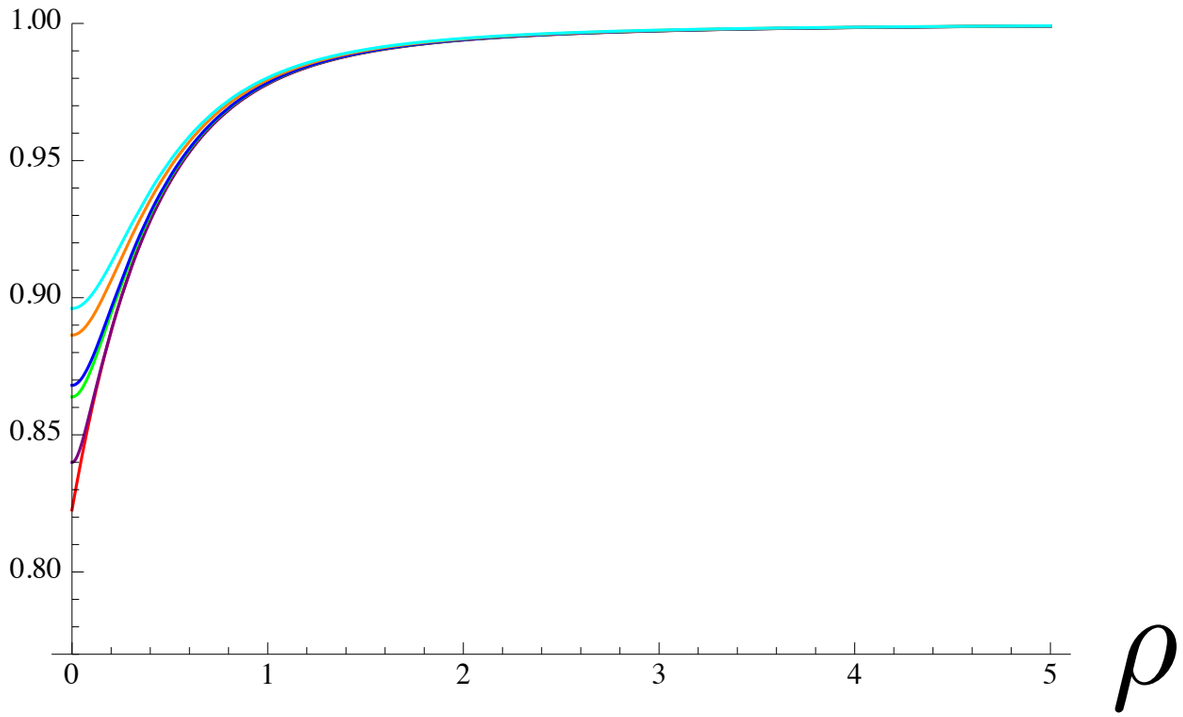
$$R^{-2}w(\rho) - m = \sum_{n=0}^{\infty} c_n e_n(\rho)$$

$$\varepsilon_n \equiv \frac{1}{2}\omega_n^2 c_n^2$$

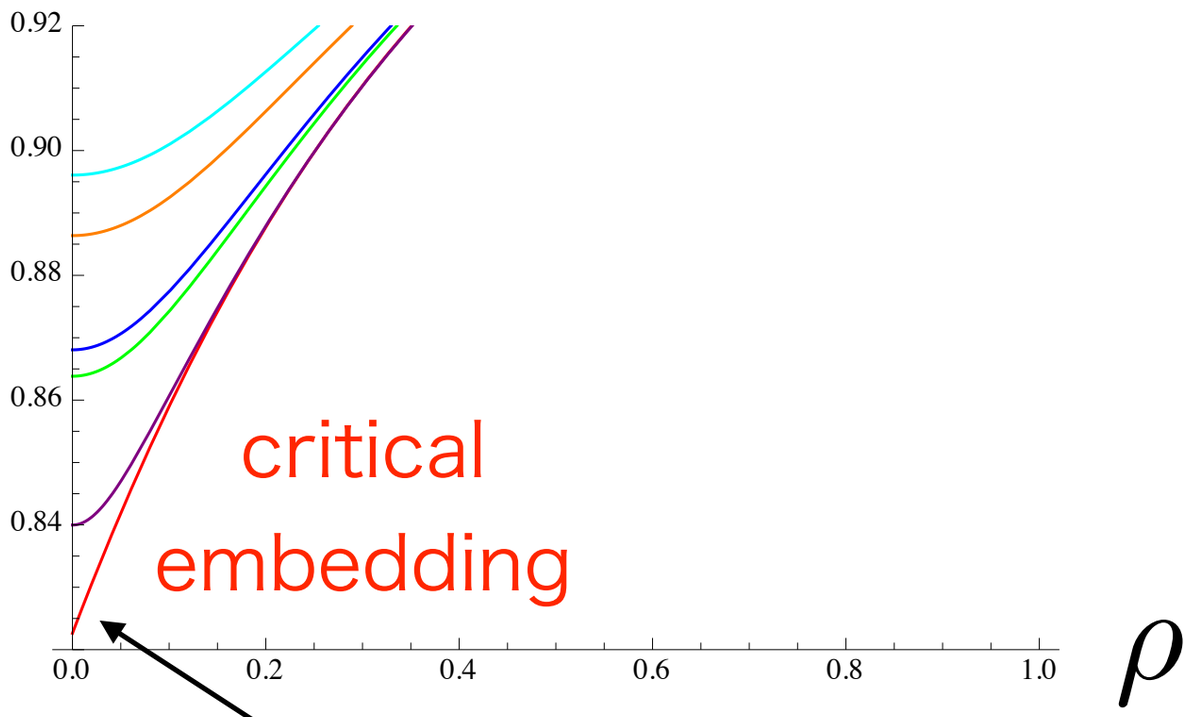
Shape of probe D-brane

D3/D7 with electromagnetic effect

$w(\rho)$



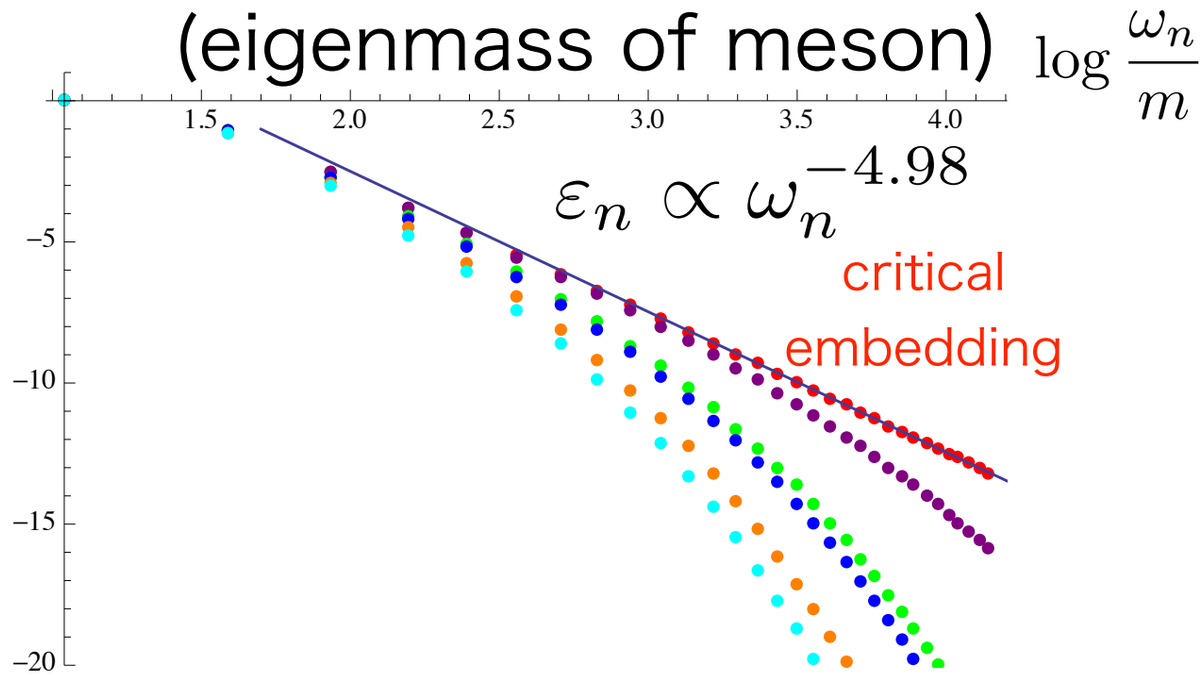
$w(\rho)$



cusp at $\rho = 0$

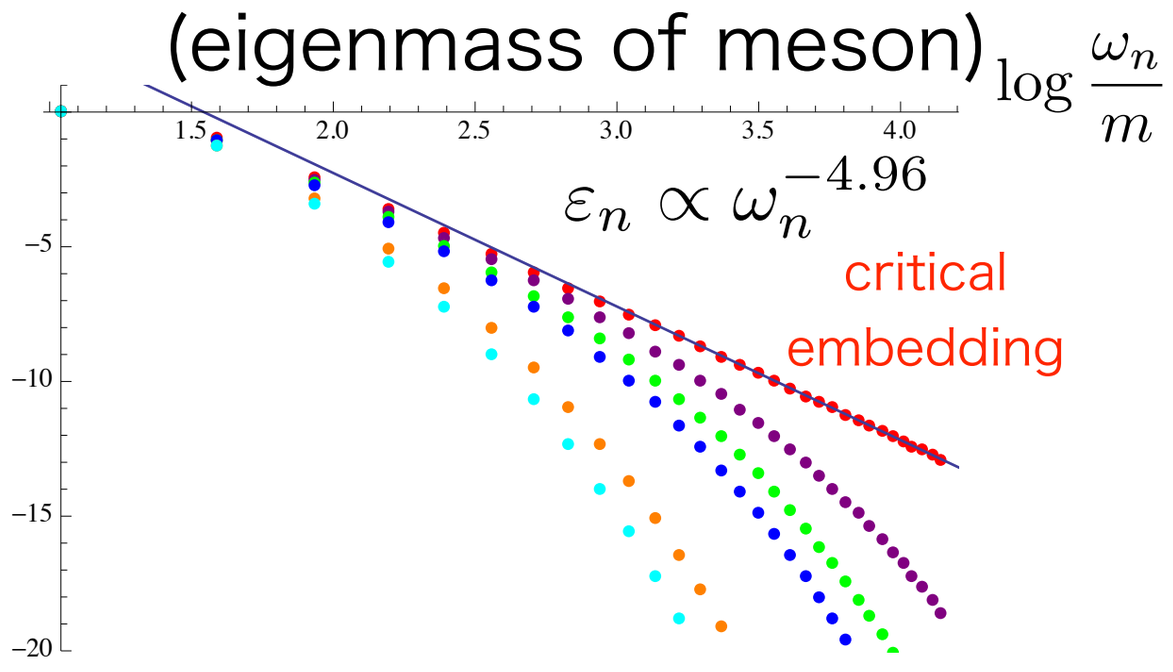
Meson energy spectrum of D3/D7

electromagnetic effect



$\log \frac{\varepsilon_n}{\varepsilon_0}$ (meson energy)

finite temperature effect



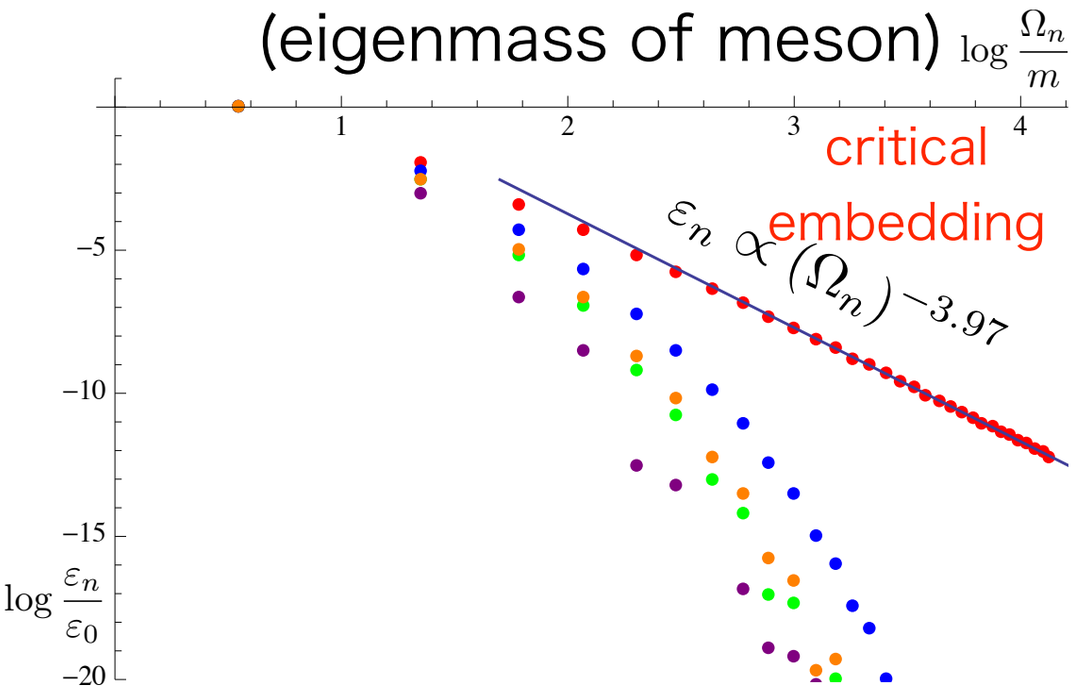
$\log \frac{\varepsilon_n}{\varepsilon_0}$ (meson energy)

D3/D7

$$\varepsilon_n \propto \omega_n^{-5}$$

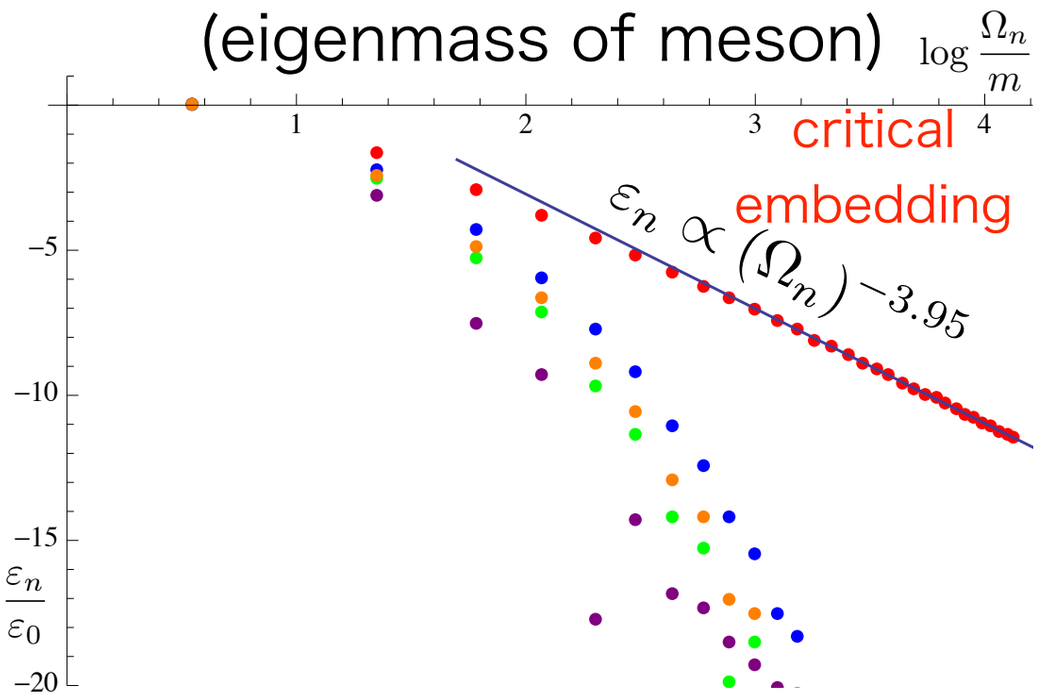
Result of D3/D5

electric effect



(meson energy)

finite temperature effect



(meson energy)

D3/D5

$$\varepsilon_n \propto \omega_n^{-4}$$

Cone dimension and power

D3/D7

$$\varepsilon_n \propto \omega_n^{-5}$$

	0	1	2	3	4	5	6	7	8	9
D3	√	√	√	√						
D7	√	√	√	√	√	√	√	√		

4

$$\left[\frac{\partial^2}{\partial t^2} - \frac{(\rho^2+R^4m^2)^2}{\rho^3}\frac{\partial}{\partial\rho}\frac{\rho^3}{R^4}\frac{\partial}{\partial\rho}\right]\chi = 0$$

D3/D5

$$\varepsilon_n \propto \omega_n^{-4}$$

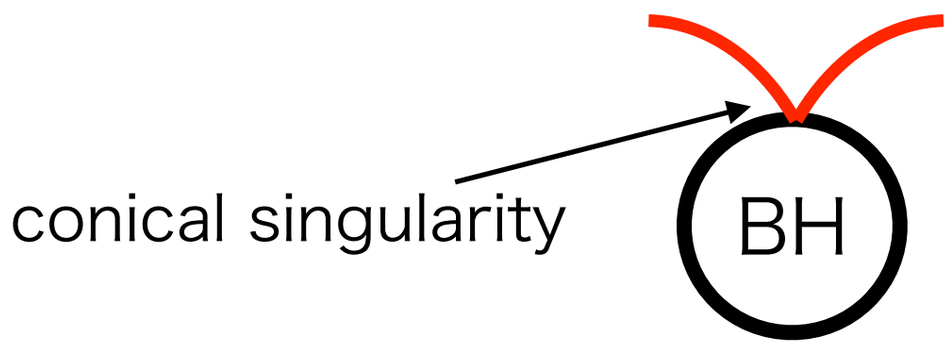
	0	1	2	3	4	5	6	7	8	9
D3	√	√	√	√						
D5	√	√	√		√	√	√			

3

$$\left[\frac{\partial^2}{\partial t^2} - \frac{(\rho^2+R^4m^2)^2}{\rho^2}\frac{\partial}{\partial\rho}\frac{\rho^2}{R^4}\frac{\partial}{\partial\rho}\right]\chi = 0$$

Conjecture

power＝－(cone dimension＋１)



Summary

- We study meson energy spectrum of D-brane systems by holography.

- We find that the power law appears in D3/D7 and D3/D5 at the critical embedding.

- The power depends on the cone dimensions of the probe D-branes.