

# Higgsing towards E-strings

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Mainly based on

- arXiv:1510.03128 "Higgsing towards E-strings"  
with Seok Kim, Kimyeong Lee

Related works:

- arXiv:1504.04614 "6d String Chains"  
with Gadde, Haghighat, Seok Kim, Lockhart, Vafa
- arXiv:1411.2324 "Elliptic Genus of E-strings"  
with Seok Kim, Kimyeong Lee, Jaemo Park, Vafa

# 6d SCFTs

- There are many 6d superconformal field theories engineered from string theory.
- Famous examples include [Witten], [Strominger], [Ganor, Hanany]
  - $N=(2,0)$  SCFT: type IIB string theory on ADE singularity. Type  $A_{N-1}$  theory describes a stack of  $N$  M5-branes.
  - $N=(1,0)$   $E_8$  SCFT describes a stack of  $N$  M5-branes, probing the  $E_8$  boundary wall of heterotic M-theory.
- Much more examples appeared in various literatures, from
  - brane systems at low energy: [Strominger], [Blum, Intriligator], [Intriligator], [Brunner, Karch], [Hanany, Zaffaroni], [Del Zotto et al.], ...
  - F-theory on non-compact CY3s: [Morrison, Vafa], [Witten], [Aspinwall], [Bershadsky, Vafa], [Heckman et al.], ...

# Self-dual strings

- 6d SCFTs contain interacting strings, which are

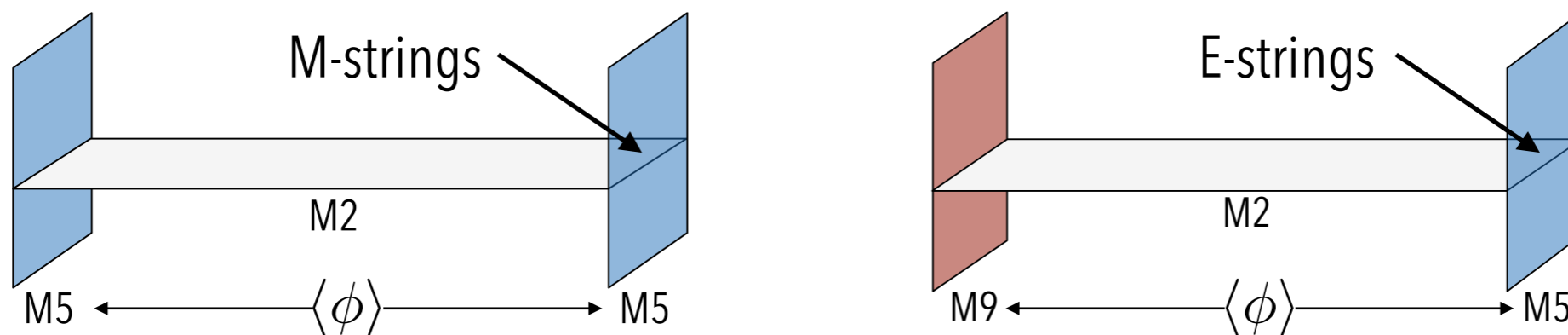
- electric & magnetic sources of tensor multiplets

$$(B_{\mu\nu}, \psi_+^A, \phi) \text{ with self-duality } H_{\mu\nu\rho} = (*H)_{\mu\nu\rho}$$

- Yang-Mills instantons (if there is a gauge symmetry)

$$k = \frac{1}{8\pi^2} \int d^4x \operatorname{tr} (F \wedge F) \in \mathbf{Z}$$

- These strings acquire non-zero tension in a tensor branch, which is parametrized by VEVs of tensor multiplet scalars.



# Higgsing chain of theories

- For a 6d gauge theory, matter contents must be tuned in order to cancel the possible gauge anomaly.

SU(3)	$N_f = 0,$	6	12	[Bershadsky, Vafa '97]
SU(2)	$N_f =$	4	10	

- ▶ They are Higgsable to M-string theory & E-string theory.

SU(3) with  $N_f = 6 \rightarrow$  SU(2) with  $N_f = 4 \rightarrow$  M-string theory

SU(3) with  $N_f = 12 \rightarrow$  SU(2) with  $N_f = 10 \rightarrow$  E-string theory

- Our aim is to study self-dual strings of SU(2) and SU(3) gauge theories which are Higgsable to M-string & E-string theories.

# Instanton moduli space

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# Instanton moduli space

- As self-dual strings are also Yang-Mills instanton solitons, their worldsheet dynamics is described by non-linear sigma model whose target space is the instanton moduli space.
- Instanton moduli space has small instanton singularity, which reflects UV incompleteness of 6d SYM. Working with singular moduli spaces is a very challenging task.

# Instanton moduli space

- Instead, we use the ADHM construction as a prescription for resolving small instanton singularity.

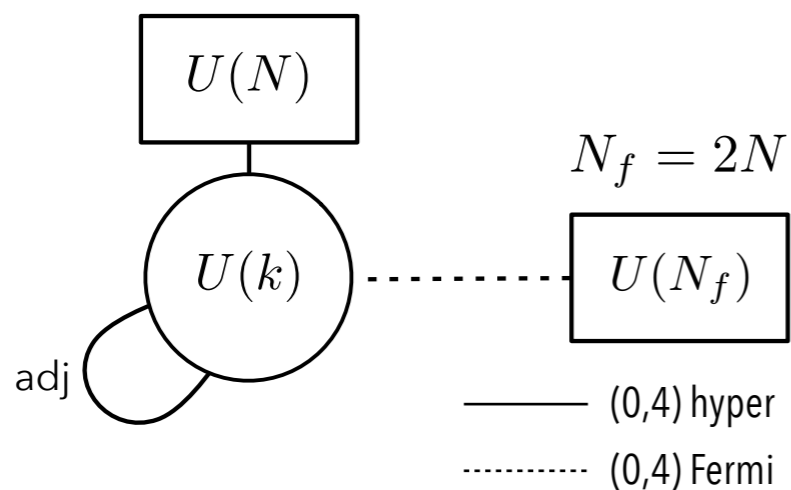
# Instanton moduli space

- Instead, we use the ADHM construction as a prescription for resolving small instanton singularity.
- The resulting ADHM gauge theory provides UV completion of non-linear sigma model in a sense that its Higgs branch recovers the instanton moduli space in IR, being away from small instanton singularity. [Witten '94]



# ADHM gauge theory

- For  $SU(N)$  gauge theories Higgsable to **M-string theory**, the ADHM gauge theory for 'k' self-dual strings is



$N=(0,4)$  SUSY generated by  $Q^{\dot{\alpha}A}$

$SU(2)_{\alpha} \times SU(2)_{\dot{\alpha}} \times SU(2)_A \times U(N) \times U(2N)$

- This theory can be engineered using 'k' D2-branes on top of 'N' D6-branes, suspended between two NS5-branes.

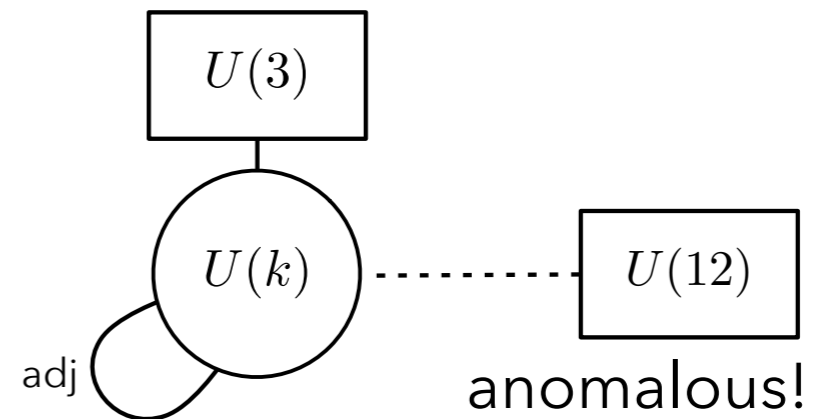
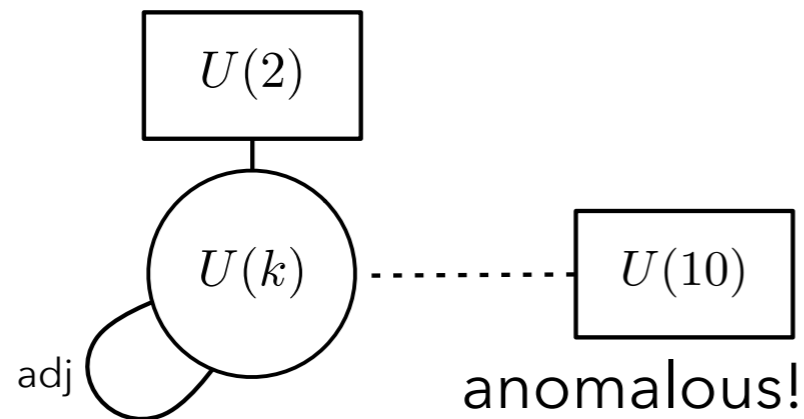


- ▶  $N=1$  case uplifts to M-string theory.

[Haghighat et al. '13]

# ADHM gauge theory

- For  $SU(3)$  and  $SU(2)$  theories Higgsable to **E-string theory**, naive ADHM construction does not yield a sensible theory.



- Hypermultiplets in

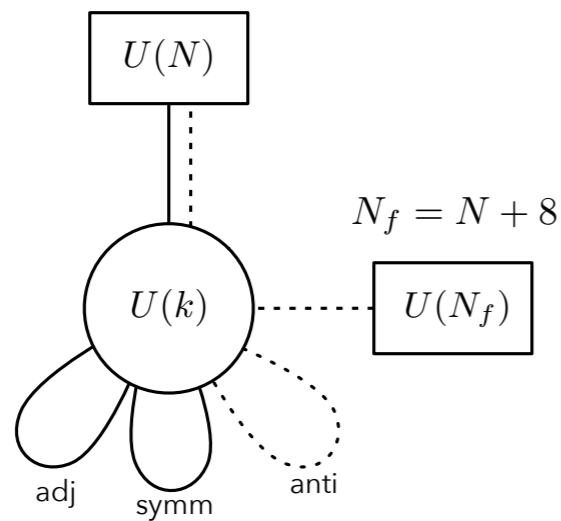
$$SU(3) : \text{ANTI}(\bar{\mathbf{3}}, \bar{\mathbf{3}}) = \mathbf{3}, \quad SU(2) : \text{ANTI}(\mathbf{2}, \mathbf{2}) = \mathbf{1}$$

representations provide different resolutions of singularity.

Having an antisymmetric hypermultiplet introduces extra degrees of freedom supported at small instanton singularity. It modifies the worldsheet gauge theory.

# ADHM gauge theory

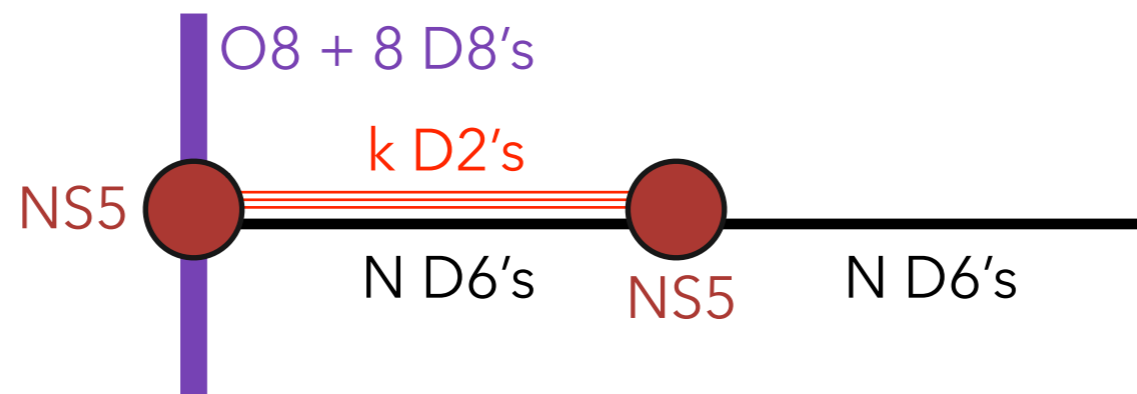
- Anomaly-free 2d gauge theory is found after interpreting  
 $6d \text{ SU}(3)$  with  $N_f = 12 \rightarrow 6d \text{ SU}(3)$  with  $N_f = 11, N_a = 1$ .  
 $6d \text{ SU}(2)$  with  $N_f = 10 \rightarrow 6d \text{ SU}(2)$  with  $N_f = 10, N_a = 1$ .



$N=(0,4)$  SUSY generated by  $Q^{\dot{\alpha}A}$

$\text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)$   
 $\times \text{U}(N) \times \text{U}(N+8) \times \text{U}(1)$

- This theory can be engineered using 'k' D2-branes in



# IR symmetry enhancement

- If our proposal is correct, the anomaly-free gauge theory should exhibit the flavor symmetries of underlying 6d QFTs. However, those are partly missing. We check if they emerge in IR by computing the elliptic genera. [Benini et al. '13]

$$\text{Tr}_{\text{RR}} \left[ (-1)^F q^{H_L} \bar{q}^{H_R} e^{2\pi i \epsilon_1 (J_1 + J_R)} e^{2\pi i \epsilon_2 (J_2 + J_R)} e^{2\pi i M A} \prod_{l=1}^{N_f} e^{2\pi i m_l F_l} \prod_{i=1}^N e^{2\pi i a_i G_i} \right].$$

$\downarrow$   
 $H_R = \{Q^{12}, Q^{21}\}$

For example, at  $k=1$ ,

$$Z_1 = - \sum_{j=1}^N \frac{\eta^{-6} \prod_{l=1}^{N+8} \theta_1(a_j - \epsilon_+ - m_l)}{\theta_1(\epsilon_1) \theta_1(\epsilon_2) \theta_1(2a_j - 3\epsilon_+ + M)} \cdot \prod_{i \neq j} \frac{\theta_1(a_i + a_j - \epsilon_+ + M)}{\theta_1(a_j - a_i) \theta_1(2\epsilon_+ - (a_j - a_i))}$$

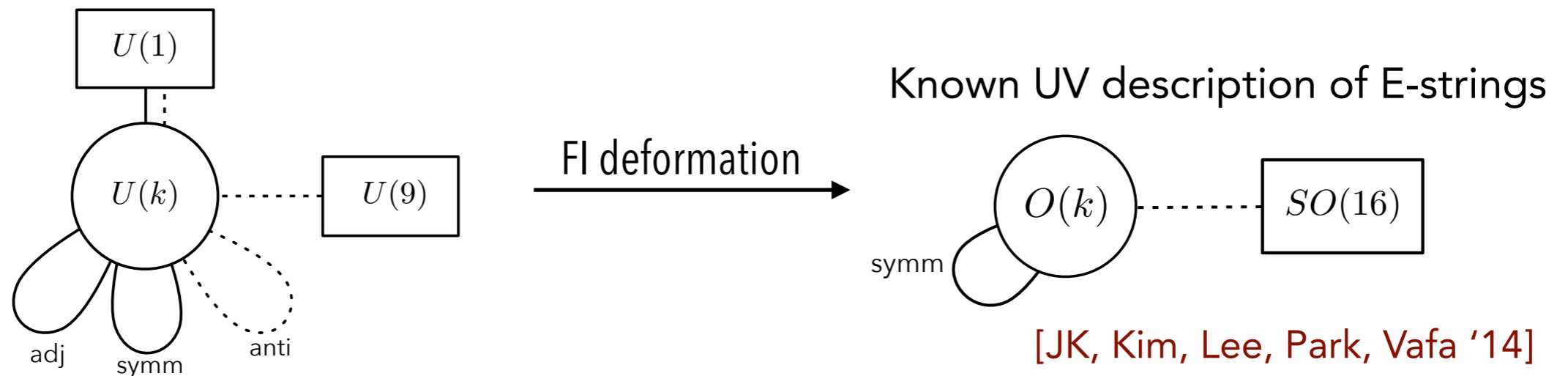
$$- \frac{1}{2} \frac{\eta^{-6}}{\theta_1(\epsilon_1) \theta_1(\epsilon_2)} \left[ \frac{\prod_{l=1}^{N+8} \theta_1(\frac{\epsilon_+ - M}{2} - m_l)}{\prod_{i=1}^N \theta_1(\frac{3\epsilon_+ - M}{2} - a_i)} + (-1)^N \sum_{I=2}^4 \frac{\prod_{l=1}^{N+8} \theta_1(\frac{\epsilon_+ - M}{2} - m_l)}{\prod_{i=1}^N \theta_1(\frac{3\epsilon_+ - M}{2} - a_i)} \right]$$

- After series expansion by  $q = e^{2\pi i \tau}$ , the BPS spectra exhibit

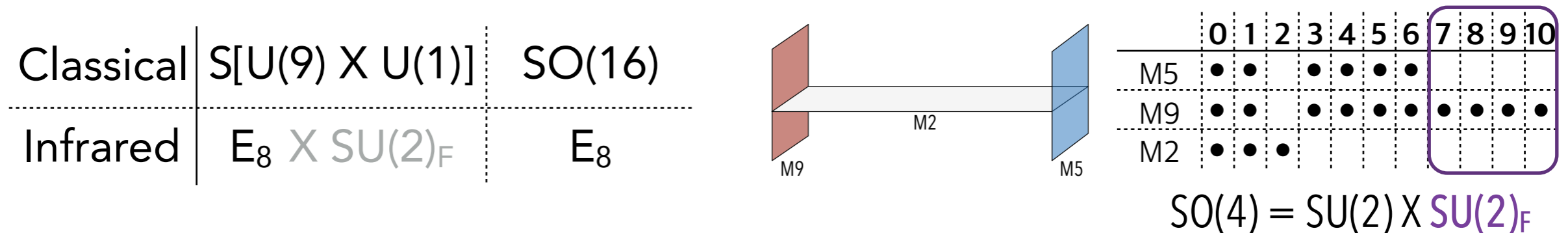
	SU(3)	SU(2)
Classical symmetry	S[U(11) X U(1)]	S[U(10) X U(1)]
IR symmetry	SU(12)	SO(20)

# Multiple M5s probing M9

- At  $N=1$ , the brane system uplifts to E-string theory where an M5-brane is near the  $E_8$  boundary wall intersecting Taub-NUT.



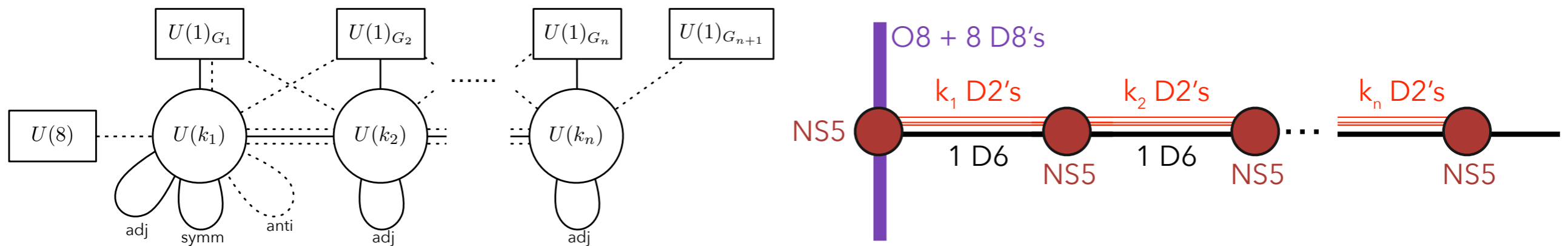
- The elliptic genus shows the IR symmetry enhancement.



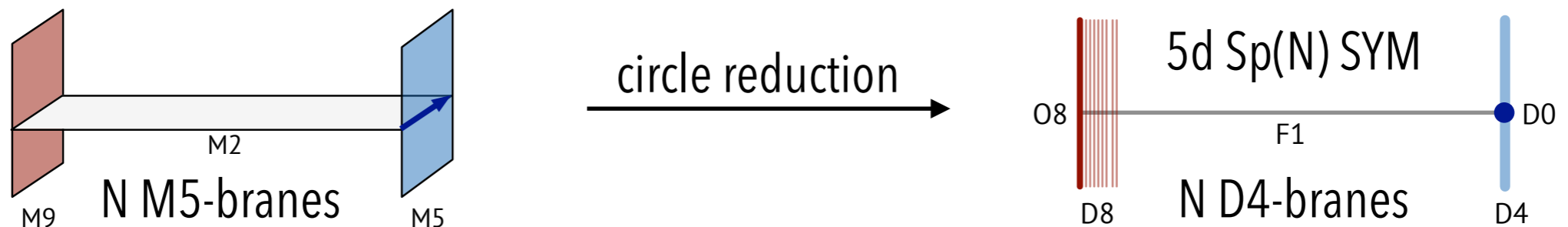
For 1 M5-brane, no BPS states are charged under  $SU(2)_F$ , so both descriptions exhibit  $E_8$  in IR and are equally useful.

# Multiple M5s probing M9

- However, one can study the fully refined BPS spectrum in case for multiple M5-branes probing the  $E_8$  boundary wall only through the left description.



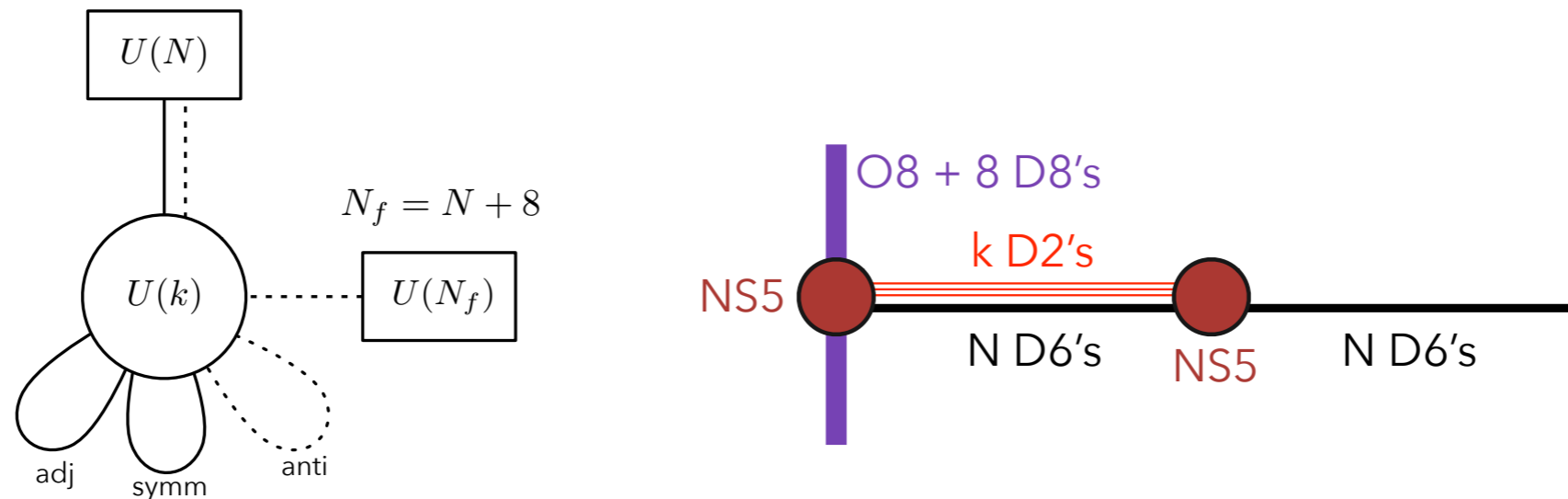
- ▶ Without  $SU(2)_F$  refinement,  $Z_{n \text{ M5}}^{\text{E-string}} = \left( Z_{1 \text{ M5}}^{\text{E-string}} \right)^n$
- ▶ We studied the BPS spectrum with 2 M5-branes, which agrees with the 5d  $Sp(2)$  instanton partition function.



[Hwang, JK, Kim, Park '14]

# Summary

- We proposed a UV description for self-dual strings in  $SU(3)$  with  $N_f = 12 \rightarrow SU(2)$  with  $N_f = 10 \rightarrow$  E-string theory



- ▶ They exhibit the correct flavor symmetry of underlying 6d SCFTs after the IR symmetry enhancement.
- This enables us to study the fully flavored BPS spectrum of multiple M5-branes probing the  $E_8$  boundary wall.